

UNCERTAIN LIFETIME, LIFE PROTECTION, AND THE VALUE OF LIFE SAVING

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ABSTRACT

The analytic innovation is treating life's end as uncertain, and life expectancy as partly the product of individuals' efforts to self-protect against mortality and morbidity risks. The demand for self-protection is modeled in a stochastic, life-cycle framework under alternative insurance options. The model helps explain the trend and systematic diversity in life expectancies across different population groups, as well as the wide variability in reported "value of life saving" estimates. The analysis yields a closed-form solution for individuals' value of life saving that is estimable empirically. It reflects the impacts of specific personal characteristics and alternative insurance options on both life expectancy and its valuation.

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Introduction

The basic objective of this paper is to analyze individuals' demand for life protection and longevity in a life-cycle context, under uncertainty concerning the arrival time of death and alternative insurance options. The significant and continuing increase in all age-specific life expectancies over the last two centuries, especially in developed countries, provides the main motivation for this study, but equally challenging is the evidence of systematic and persisting diversity in the life expectancies of different population groups. A related puzzle has been the widely diverse estimates of the private 'value of life saving', based on alternative regression estimates of 'willingness to pay' for a marginal reduction in mortality risks (for a recent survey, see Viscusi, 1993). This paper seeks to address and link both of these diversities through an explicit life-cycle model of the demand for life expectancy and the value of life saving.

The model combines elements of the analysis of optimal insurance and self-protection in Ehrlich and Becker (1972) (henceforth EB), and the life-cycle model of consumption and bequest choices under uncertain lifetime in Yaari (1965). The model treats the risk of mortality as an object of choice in a stochastic, dynamic setting in which individuals determine their consumption, life protection, and bequest plans at any given age, subject to survival to that age. The demand for longevity is thus identified as that for reducing the risks of mortality and morbidity over the life cycle, i.e., for lifetime 'self-protection' in the EB terminology. The mechanism for self-protection involves health promotion activities and related life-style and occupational choices, which lower mortality and morbidity risks over the life cycle.

In this setting, the existence of complete markets for insurance (actuarial notes) can be utilized by individuals to insure separately against "living too long", by purchasing guaranteed annuities, and "living too short", by purchasing conventional life insurance. Ordinary savings constitute an alternative means of self-insurance against both hazards, but since such savings are inherently transferable to survivors

(unlike annuities), they insure the two hazards jointly rather than separately. The analysis concerns the interaction between self- protection and all three insurance options over the life cycle.

This framework produces a theory of the demand for life expectancy under uncertainty, which complements the life-cycle model of the demand for life extension under certainty in Ehrlich and Chuma (1987, 1990). As in the earlier work, the analysis focuses on both the demand for life protection and a "dual" variable associated with it -- the shadow price of the probability of survival, better known as the 'value of life saving' in the extensive literature spawned by Schelling (1968) and Mishan (1971). The shadow-price function sheds new light on the determinants of the value of life protection, and hence the demand for it. Under actuarially fair insurance terms and some simplifying assumptions, the analysis yields a closed-form solution for the value-of-life-saving function that can be implemented empirically through calibrated simulations or econometric analysis. The solution incorporates as a special case the net discounted value of future labor income, which served as a conventional "value of life" measure in the early literature on human capital.

The paper generalizes previous analyses of the private value of life saving by Thaler and Rosen (1975), Conley (1976), Shepard and Zeckhauser (1984), and Rosen (1988). Unlike these previous contributions, most of which were developed under static conditions, the present analysis is pursued in a dynamic, stochastic setting, which allows for continuous revisions of optimal consumption and bequest plans along with self-protection choices over the life cycle. By modeling conditional mortality risks as outcomes of self-protection and its interaction with market insurance options, the model explores the impact of insurance on life protection. It also exposes the relative impact of human versus non-human wealth, and of life insurance versus annuities, on life expectancy and the value of life saving.

The analysis shows that both life expectancy and the implicit value people place on staying alive vary systematically with identifiable parameters. These include endowed wealth, "natural" mortality risks,

age, earning capacity, the rates of interest and time preference for consumption, medical technology and costs, relative bequest preferences, and alternative market (or social) insurance options. When complete markets for actuarial notes are available, these same parameters determine the consumption path and bequest as distinct choice variables, and hence, indirectly, optimal life insurance and annuities. These two types of insurance are shown, in turn, to have opposing effects on optimal life protection. The analysis assumes for the most part that insurance terms, if available, are actuarially fair, fully reflecting individual mortality risks, but it also considers cases (especially in sections 2 and 5) in which insurance terms are actuarially unfair or prohibitively high.

The model thus rationalizes, within predictable bounds, both the trends and the apparent sizeable diversity in life expectancies across different population groups and countries in different stages of development. Moreover, it provides a theoretical foundation for rationalizing the apparent huge diversity in empirical estimates of values of life savings, based on the ‘willingness to pay’ approach. A summary of the model’s main inferences concerning the interaction between insurance, life protection, and the value of life saving, as well as the role of alternative insurance options in affecting life expectancy and related life-time choices, is given section 7.

1. Basic Framework

Unlike models of the demand for health and longevity where the time of death is treated as known with certainty, this paper recognizes mortality, and the attendant incidence of morbidity or injury, as stochastic, thus possibly imminent events. The incidence of these events is dictated largely by biological or environmental factors. The model’s basic assumption, however, is that it can also be controlled on the margin through health-and safety-enhancing efforts. Choosing an optimal amount of the latter can be referred to as the **protection** problem.

The **insurance** problem associated with uncertain life spans is twofold: it concerns coverage of uncertain consumption needs associated with living too long (“old age insurance”) or living too short (survivors benefits, or “life insurance”). Depending upon the existence of insurance markets and bequest motives, these hazards can be controlled jointly through ordinary savings, or separately through the purchase of actuarial annuities and a mix of conventional life insurance and ordinary savings.

The incidence of mortality is modeled as a continuous Poisson process. By this process, the probability of survival between any two periods a and b is expressed by the exponential distribution law:

$$p(a, b) = \exp(-m(a, b)), \quad \text{where } m(a, b) = \int_a^b f(t) dt ,$$

and $f(t)$ represents the conditional probability density of the occurrence of death at t , given survival to that date. The basic assumption is that the arrival frequency of death or “force of mortality”, $f(t)$ (see Borch, 1978), can be controlled via a flow of self-protective devices $I(t)$ as follows:

$$(1.1) f(t) = j(t) - I(t) \geq 0 , \quad \text{with } I(t) = I(m(t), M(t); E'(t), t).$$

In equation (1.1) $j(t) > 0$ stands for the “endowed” conditional density of mortality, as determined by natural or biological factors. Its rate of change is assumed to be positive at all points in time, or $\overset{o}{j}(t) \equiv dj(t)/dt > 0$.

By equation (1.1) the conditional probability of survival through a specific age, while being a decreasing function of age, is also controllable through self-protective services, $I(t)$, identified as the difference between the biological and actual mortality risks, $[j(t)-f(t)]$.¹ These services are produced through inputs of time, $m(t)$, market goods, $M(t)$ (having a unit price P), and efficiency parameters such as education, $E'(t)$, or age, t .

The production function is assumed to be subject to diminishing returns to scale in view of the constancy of the human body. This assumption, along with the concavity of the utility and healthy-time

function (see equations 1.2 and 2.2 below), guarantees that the model has interior dynamic solutions (cf. Ehrlich and Chuma, 1990). It also implies that life protection activity is subject to increasing marginal costs. A general specification of the cost function is thus given by $C(I(t)) = c(t)I(t)^\alpha$, where $\alpha > 1$ and $c(t) = c(w(t), P(t), E'(t), t)$, where $w \equiv w^0(E')/E'$, the observed wage rate per unit of human capital, indicates the opportunity cost of efficiency time. The analysis assumes, for simplicity, that $\alpha = 2$, and that all prices and other efficiency indicators remain constant over the life cycle. The presumed constant “price” of efficiency time, w , is justifiable on the assumption that human capital accumulation over the life cycle has a “neutral effect” on both the observed personal wage rate and the person’s efficiency in health production. The life protection cost function thus becomes $C(I(t)) = cI(t)^2$.

Although mortality and morbidity risks can, in principle, be separately controlled, they are viewed in the first part of the paper as monotonically related. The force of mortality may thus serve as a general index of health. Since morbidity or injury typically diminish the time available for productive purposes, “healthy time” is expressed as a decreasing and concave function of $f(t)$:

$$(1.2) \quad h(t) = h(f(t), \beta), \text{ with } h'(\cdot) < 0 \text{ and } h''(\cdot) < 0,$$

where β represents technological innovations that decrease the amount of sick time associated with episodes of morbidity. Healthy time and sick time together exhaust the instantaneous time constraint implicit in the analysis, Ω . For convenience, consumption activity $Z(t)$ is assumed to be produced via inputs of time and market goods at constant unit costs.

Given the uncertain prospect of mortality, there is a distinction between the actual (ex-post) arrival time of death, which is a stochastic variable, \tilde{D} , and the ex-ante planning horizon, which is a determinate one, $D \leq \infty$. The simplest assumption would be that D is infinite. The lifetime optimization problem can be simplified analytically, however, by recognizing that beyond a certain finite planning horizon, D , individuals' choices would become invariant to the planning horizon’s length, essentially

because of the intensifying force of mortality.² The relevant expected lifetime utility functions are stated in equation (2.2) and (4.4) below. In these equations E stands for the expectation operator concerning the stochastic length of life, \tilde{D} , ρ denotes the subjective discount rate, and the instantaneous utility function U is assumed to be concave.

Because of the complexity of the full-fledged protection and insurance problem, we analyze it in stages. Self-protection is first considered without any insurance options or a bequest motive (section 2). Next, an annuities market is introduced (section 3) in which the unit cost of insurance is generally assumed to be actuarially fair at the individual level. The distinct role of bequest and life insurance versus annuities is then explored in the more general case of sections 4. In section 5, the actuarially fair insurance assumption is relaxed, and a comparison is made between optimal life-protection choices with and without complete markets for annuities and life insurance. Table 1 in the appendix presents a glossary of the model's key variables.

2. Self-protection in the Absence of Insurance Markets

To derive basic insights into the protection problem, it is assumed, first, that no organized insurance markets are available, and there is no distinct bequest motive. Apart from the ability to engage in self-protection, a person can also self-insure against financial hazards stemming from an uncertain lifetime by accumulating ordinary savings. Since accumulated savings are transferable to survivors, their stock at time t , $A(t)$, represents a potential “accidental bequest” level in this case.

The instantaneous wealth constraint limiting protection and consumption choices is given by

$$(2.1) \quad \overset{0}{\dot{A}}(t) = rA(t) + wh(f(t)) - cI^2(t) - Z(t),$$

where $\overset{0}{\dot{A}}(t) \equiv dA(t)/dt$ represents the rate of change of accumulated assets at t , or current savings, as a

residual of income net of consumption and self-protection costs. The parameters r and w denote the constant rate of interest and wage rate, respectively, $h(t)$ represents healthy labor time, and the full price of consumption is chosen as a numeraire ($P_Z = 1$). Note that $wh(t) - cI^2(t)$ represents potential labor income (as a function of total healthy time), net of life-protection costs, including the opportunity cost of time spent on health maintenance.

The wealth constraint must be further restricted, however, by the requirement that $A(t) \geq 0$ for all t in $[0, D]$. This is because without a market for insurance, the institutional framework of the capital market makes it virtually impossible for a person to die with a negative net worth (cf. Yaari, 1965). To avoid any discontinuities in the state and co-state variables of the model, which arise whenever $A(t)$ assumes its boundary value, the optimization analysis is pursued only for the case where $A(t) > 0$ in all periods of life.

Absent a bequest motive, the optimal life protection and consumption plan must satisfy the maximized (selfish) expected lifetime utility function:

$$(2.2) J(A(t), t; \mathbf{a}) = \text{Max}_{Z, I} E \left\{ \int_t^D \exp[-\mathbf{r}(s-t)] U(Z(s), h(f(s))) ds \right\},$$

where E stands for the expected utility operator. Expected utility is maximized subject to constraints (1.1), (2.1), $A(t) > 0$, the terminal conditions $A(D) \geq 0$ and $J(A(D), D; \alpha) = 0$, and the vector of exogenous parameters $\alpha \equiv w, E, P, \rho, r, j$. Applying the stochastic dynamic programming approach (see, e.g., Judd, 1998), the optimal solution, $\{Z^*(t), I^*(t)\}$, is determined by the Hamilton-Bellman-Jacobi condition:

$$(2.3) -J_t = -(\rho + f^*)J + U(Z^*, h(f^*)) + J_A[rA + wh(f^*) - cI^{*2} - Z^*]$$

where $J_t \equiv \partial J(A(t), t; \alpha) / \partial t$, and Z^* and I^* satisfy the optimality conditions

$$(2.4) U_Z(Z^*, h(f^*)) = J_A$$

$$(2.5) \quad 2cI^* = J/J_A + [w + (1/J_A)U_h(Z^*, h(f^*))][-h'(f^*)] \equiv v_0^* .$$

2.a. The dynamic paths of protective outlays and consumption

Equation (2.5) expresses the equilibrium condition for optimal self-protection.³ The L.H.S. represents the marginal cost of reducing the force of mortality, and the R.H.S. the shadow price of that reduction, or the value of self-protection. In this formulation, however, the **full** value of self-protection is comprised of two distinct components. The first is the value of the remaining life span (J/J_A), which may be called 'the value of life protection, or life saving', using Schelling's terminology. The second represents 'the value of health saving', i.e., that of the added amount of healthy time arising from the marginal increase in self-protection. The second component emerges if self-protective endeavors are assumed to effect a reduction in both mortality and morbidity risks, and hence a decrease in the opportunity costs of ill health.

From equation (2.5) it can further be shown that if the utility function is separable in consumption and healthy time, the dynamic path of self-protective expenditures, and hence the proportionally related value of life protection, would be given by

$$(2.6) \quad I^*(t) = (1/\Delta) \{ [d(J/J_A)/dt + (U_h/J_A)(-h')(r - \rho - f^*(t))] \\ - [w + (U_h/J_A)h' + (U_{hh}/J_A)(h')^2] j^*(t) \} = v^*(t)$$

where $\Delta \equiv 2c - [w + (U_h/J_A)h' - (U_{hh}/J_A)(h')^2] > 0$, and $\dot{X} \equiv dX/dt$ for any variable, X..

The path of health investment thus depends on two competing influences: (1) the rate of "aging", or the biological rate of increase in the probability of morbidity $j^*(t)$, which produces a positive impetus for investment, as the marginal utility from falling healthy time rises with age; (2) the change in the value of reducing mortality and morbidity risk $d(J/J_A)/dt + (U_h/J_A)(-h')(r - \rho - f^*(t))$, which produces a mixed

impetus: while the second term in this expression is positive, because of the incentive to defer all consumer spending (see below), the first term is indeterminate (unless f^* becomes sufficiently large with age) except at the last phase of the planning horizon where the change in the value of the remaining life span $d(J/J_A)/dt$ becomes negative, since the value function itself approaches its terminal value $J(D) = 0$. Conceivably, the second term in (2) may dominate the first, except at the most advanced stages of the planning horizon.

The analysis indicates the way the value of life and health protection changes over the life cycle. Although under uncertainty, death or injury may be imminent rather than distant future prospects, the value of life and health protection is seen to be rising over a good part of the life cycle because aging raises the benefits from protection except in late phases of the planning horizon. Private expenditure on health and life protection may thus be rising even during the last phase of the **actual** life span of most persons, as available data generally show. The reason is that the actual time of death must generally occur **prior** to the end point of the planning horizon, where the value of health protection becomes minimal and the value life protection is zero.

The consumption path is similarly derived from equation (2.4):

$$(2.7) \quad \dot{Z}^0(t) = -[U_Z(t)/U_{ZZ}(t)](r - \mathbf{r} - f^*(t)) - [U_{Z_h}(t)/U_{ZZ}(t)]h'(f^*(t))f^{*0}(t).$$

Clearly, if utility were separable in consumption and healthy time ($U_{Z_h}(t) = 0$), equation (2.7) would reaffirm the well known result that in a world without insurance, consumption would continually rise over the life cycle only if the market discount rate exceeded both the subjective discount rate and the conditional risk of death (see Yaari 1965). An exogenous reduction in f will then increase the incentive to provide for future consumption. This result has an interesting interpretation in terms of the incentive to "self-insure" the consumption needs associated with 'living too long'. A reduction in mortality risk implies a higher probability of survival to old age. It therefore increases the incentive to secure old-age

needs, as is generally the effect of an increased probability of a hazard on the incentive to self-insure against it (see EB, 1972). As in the case under certainty (see Ehrlich and Chuma, 1990), however, equation (2.7) further implies that if consumption and healthy time are complementary goods ($U_{zh}(t) > 0$), then consumption is more likely to fall at old age, since healthy time decreases due to the rising risk of morbidity.

2.B. Comparative Statics Predictions

Behavioral predictions are derived here under two conditions: (1) the state variable $A(t)$ is taken to be predetermined, given survival to time t ; and (2) the direction of change in the maximized expected utility J and in the expected marginal utility of wealth J_A caused by any parameter change are assumed a priori. The second assumption is invoked to facilitate an otherwise intractable analysis of comparative dynamics, because equation (2.5) does not have an explicit solution. The resulting comparative-static predictions are summarized in Table 2.⁴

It is interesting to note that these behavioral predictions corroborate analogous predictions derived under conditions of certainty concerning the timing of death (see Ehrlich and Chuma, 1990, Table 3). Thus an increase in $\alpha = A(t)$, w , E' , or β is assumed to lead to an increase in life-time wealth (consequently, $J_{A\alpha} < 0$), and hence in the demand for, and value of, life protection (provided a partial increase in the market wage rate by itself, with no change in human capital, does not raise significantly the full unit cost of life protection or consumption). An increase in $\alpha = P$ or ρ , by contrast, is expected to lead to the opposite changes. An increase in the rate of interest, r , will here lead to unambiguous increases in the demand for self-protection and consumption as long as the individual is a net saver, since it would then generate an increase in life-time wealth without affecting the cost of health protection.

An increase in time preference, in contrast, decreases the incentive to postpone consumption (self-insure) as well the demand for self-protection.

The ambiguous impact of an exogenous increase in the biological risk of mortality and morbidity, j , on self-protection, is the result of possibly opposing effects. The ambiguity stems, in part, from the unknown impact of higher risk levels on productivity at self-protection. An increase in these risks also has an adverse effect on wealth, which lowers the value of life saving, but it also raises the marginal benefit from a unit of healthy time. To the extent that the increase in j represents just an increase in mortality risk, however, it may reduce the demand for self-protection and its value, as well as the incentive to self-insure future consumption through savings.

The comparative-statics predictions concerning the demand for self-protection $I^*(t)$ apply equally well to the demand for life expectancy, which is defined by

$$T^*(t) = \int_t^D \exp[-m(t,u)]f(u)(u-t)du, \quad \text{where } m(t,u) \equiv \int_t^u f(s)ds,$$

and $\exp[-m(t,u)]f(u)$ is the probability that a person of age t will die at age u . The analysis thus yields a set of indirect, age-specific demand functions for life expectancy, T^* , self-protection, I^* (or health $h(f^*)$), consumption, Z^* , and value of life saving, v_o^* , of the following specification:

$$(2.8) \quad F(t) = X(t, A(t), P, E', w, j, r, \rho, \beta), \quad \text{where } X \text{ stands for } T^*, I^*, Z^*, \text{ and } v_o^*.$$

The properties of this set of functions are listed in Table 2.⁵

3. Adding a Market for Annuities

The opportunity set underlying life protection decisions changes markedly when we allow for transactions in actuarial notes between individuals and insurance companies. Such a transaction takes place, for example, if a person buys an annuity that stipulates that the owner receive a certain interest income from the company as long as she lives, and at a rate higher than the market rate of interest for

regular notes, but nothing if she dies. The converse transaction -- selling an actuarial note -- is equivalent to permitting a person to borrow against his potential life-time earnings, but freeing his estate from any obligation in the event of his death. Like ordinary savings, annuities support the desired path of old-age consumption as long as a person is alive. The unique "insurance" offered by annuities is avoidance of assets left as accidental bequest, for which the insured is compensated in the form of an extra return for life over ordinary assets.

In the absence of any bequest motive, market insurance through annuities will dominate self-insurance through conventional savings since the return is higher on the former while the opportunity costs are the same for both (see Yaari, 1965). In this case, all savings, and obviously any borrowing against future labor income, would be exclusively through actuarial notes.

If such insurance transactions can be effected at actuarially fair terms, a person will be able to borrow his expected future net labor income or lend his accumulated savings in period t at equal terms, $(r+f(t))$, representing the sum of the return on regular notes plus an interest premium equal to the force of mortality. The effective net worth at period t thus becomes

$$A_1(t) = \exp(rt + m(0, t)) \left\{ A_1(0) + \int_0^t \exp(-ru - m(0, u)) [wh(u) - cI^2(u) - Z(u)] du \right\},$$

where $m(0, u) = \int_0^u f(s) ds$, so the instantaneous wealth constraint must be restated as

$$(3.1) \quad \dot{A}_1(t) = (r + f(t)) A_1(t) + wh(f(t)) - cI(t)^2 - Z(t).$$

The relevant optimal value function, in turn, becomes

$$(3.2) \quad J_1(A_1(t), t; \mathbf{a}) = \text{Max}_{Z, I} E \left\{ \int_t^{\tilde{D}} \exp(-\mathbf{r}(s-t)) U(Z(s), h(f(s))) ds \right\},$$

subject to (1.1), (3.1), $J_1(A_1(D), D; \alpha) = 0$, and $A_1(D) \geq 0$.

The basic modifications in previous results concern the optimality conditions for self-protection and

consumption. The one for self-protection becomes

$$(2.5a) \quad 2cI^* = [J_1/J_{1A} - A_1] + [w + (1/J_{1A})U_h(t)][-h'(f^*)] \equiv v_1^* .$$

Equation (2.5a) implies that the shadow price of the risk of survival, or value of life saving, is now dependent, in part, on whether the individual is a net borrower or a net lender of assets (cf. Conley, 1976). The rationale is that since an investment in self-protection by the investor reduces the risk premium, and thus the rate of return on lending, the investor will realize a capital loss in the amount of the accumulated assets. It would appear that this factor would induce a shift in the optimal timing of investment in self-protection towards periods in which the investor is a net borrower of funds. This conclusion would be valid, however, only for a "compensated" change in borrowing status that does not affect expected wealth. Indeed, the comparative statics analysis of the effect of wealth applicable to this section (see below), as well as the explicit solution for the optimal value function $J_1(t)$ in the following section, demonstrate that a larger net assets position always increases the value of life saving

The time path of self-protection under the annuities option remains much the same as the one derived in the preceding section, with the exception that self-protection activity is expected to rise more steeply with age.⁶ As for the optimal path of consumption, equation (2.7) becomes

$$(2.7) \quad \dot{Z}^0(t) = -[U_z(t) / U_{zz}(t)](r - \mathbf{r}) - [U_{zh}(t) / U_{zz}(t)]h'(f^*(t)) \dot{f}^{*0}(t) .$$

This path is similar in shape to the one derived under conditions of certainty (see, e.g., Ehrlich and Chuma (1990), eq. 19). Actuarially fair insurance effectively eliminates the influence of the risk-discount factor for uncertain future we have encountered in equation (2.7). It increases the motivation to delay consumption, i.e., provide insurance against "living too long", essentially because the discount factor for uncertain survival is fully compensated for by an equal increase in the rate of return on annuity savings while being alive.

This conclusion would change, of course, if market insurance were actuarially unfair at the individual

level because of transaction or monitoring costs. Such costs require in principle that **opposite** loading terms be charged on borrowing and saving transactions, and a dynamic program concerning optimal consumption and life protection decisions must then be derived conditionally for cases in which a person is strictly a net saver or borrower. Since a positive loading, q , reduces the return on annuities to $r + f(t)(1-q)$, some discounting for the risk of mortality would still be present in equation (2.7a), lowering the slope of the optimal consumption path, as is the case in equation (2.7). No other modifications apply in the preceding analysis except for the recognition that a positive loading factor would now play a role similar to that of a reduced market interest rate (for net savers) in Table 2. The loading factor will lower the incentive for life protection, as is generally the effect of insurance loading on self-protection (see EB, 1972).

The availability of a market for actuarial notes does not alter, however, the qualitative comparative-statics predictions of section 2B essentially because these have been conditional on a realized magnitude of current wealth. This conclusion is easily shown to apply to the effects of a change in current wealth as well. Table 2 thus generally applies to this section as well.

The existence of a market for actuarial notes, however, does exert an independent influence on the value of life saving, as is apparent from a comparison of the first components of equations (2.5) and (2.5a), representing the willingness to pay for a marginal reduction just in the risk of mortality. The direction of that influence, seemingly ambiguous at this point, will be clarified in section 5 below, following our more general analysis of the issue in the next two sections.

4. The Model with Complete Insurance Markets and Bequest Preferences

The market for actuarial notes creates opportunities to insure separately the financial hazards associated with "living too long" and "living too short". A positive demand for life insurance requires,

however, the existence of a bequest motive or altruism toward dependents.

As in the preceding case of insurance against living too long, insuring dependents against the consequences of one's premature death can be achieved through either market insurance (i.e., conventional life insurance) or self-insurance through ordinary savings, which are inherently transferable to survivors. In this context, however, neither form of insurance dominates the other. Indeed, both are subject to **identical** opportunity costs on the margin, since life insurance policies are purchased implicitly by selling an actuarial note and buying instead the right to a regular saving note (see Yaari, 1965). Thus, the mix of life insurance and regular savings is in principle indeterminate. The critical individual choice concerns the **sum** of the two, i.e., the optimal amount of bequest or "total life insurance".

To facilitate the derivation of an analytical solution for this general, but more complex case, it is necessary to introduce some simplifications outlined below. The solution generalizes earlier specifications of the comparably defined "value of life saving".⁷

4.A. Introducing Bequest

Self-protection is assumed to affect exclusively mortality risk. The impact of protection on morbidity risk, hence current earnings or utility, is thus ignored. Although designed to simplify the analysis, this assumption is in line with the literature on the value of life saving, which has generally ignored morbidity risks. The analysis of section 3 is generalized in an important way, however, by incorporating in the objective function a utility-of-bequest function, $V(B,t)$ with the following properties:

$$(4.1) \quad V(0,t) \leq 0; \quad V_B(B,t) > 0, \quad V_{BB}(B,t) < 0, \quad B \geq 0,$$

where B stands for the amount of bequest.

Under actuarially fair pricing of personal annuities and life insurance, the force of mortality ($f(t)$)

represents the unit opportunity cost of regular saving and life insurance (hence bequest) as well as the unit premium on annuities. The instantaneous wealth constraint becomes

$$(4.2) \quad \dot{A}_1 = (r+f(t))A_1(t) + wh(t) - cI^2(t) - Z(t) - f(t)B(t),$$

where A_1 represents here the sum of accumulated ordinary savings and annuities, or financial wealth, subject to the restriction that a person cannot reach the end point of the planning horizon with negative financial wealth, or $A_1(D) \geq 0$, by the institutional constraints on actuarial notes. These constraints also restrict bequest not to exceed the person's total wealth, i.e.,

$$(4.3) \quad 0 \leq B(t) \leq A_1(t) + L(t) \geq 0,$$

where $L(t)$ denotes the discounted value of potential net life-time earnings stream, or human wealth.⁸ They further require that total wealth cannot be negative, as borrowing cannot exceed human wealth.

The maximized expected utility function in this formulation is given by

$$(4.4) \quad J_1(A_1(t), t; \mathbf{a}) = \text{Max}_{Z, B, I} E \left\{ \int_t^{\tilde{D}} \exp[-\mathbf{r}(s-t)] U(Z(s)) ds + \exp[-\mathbf{r}(\tilde{D}-t)] V(B(\tilde{D}), t) \right\},$$

subject to equations (4.2) and (4.3). Again, by the stochastic dynamic optimization approach, the optimal solution must satisfy

$$(4.5) \quad -J_{I1} = -\rho J_1 + U(Z^*) + J_{1A} [(r+f)A_1 + wh - c(I^*)^2 - Z^* - fB^*] \\ + f[V(B^*, t) - J_1],$$

subject to the boundary conditions $J_1(A_1(D), D; \alpha) = V(B^*(D), D) = V(A_1(D), D)$ and $A_1(D) \geq 0$. The optimal control variables I^* , Z^* , and B^* are then solved from

$$(4.6) \quad 2cI^* = (1/J_{1A}) [J_1(A_1(t), t; \alpha) - V(B^*)] + B^* - A_1 \equiv v_1^*(t)$$

$$(4.7) \quad U_Z(Z^*) = J_{1A}$$

$$(4.8) \quad B^* = \begin{cases} 0 & \text{if } J_{1A} > V'(B^*) \\ \varepsilon(0, A_1(t)+L(t)), & \text{if } J_{1A} = V'(B^*) \end{cases}$$

$$A_1(t)+L(t), \text{ if } J_{1A} < V'(B^*).$$

A corner solution for optimal bequest, and hence for total life insurance, is obtained where the marginal value of bequest either falls short of or exceeds the expected marginal utility of wealth over the remaining life span. A no-bequest, thus no-life-insurance solution, is more likely, of course, if concern for dependents is small, as may be the case for unrelated individuals. But in this case, the existence of the self-protection alternative will be shown to increase the person's demand for it. An interior solution for bequest and life insurance is the more representative case, however, and the following analysis focuses on this case and its behavioral implications.

4.B. Explicit Solutions

To obtain an explicit solution for the maximized life-time expected utility function J_1 in equation (4.5) the instantaneous utility of consumption function is specialized to exhibit 'constant relative risk aversion,

$$(4.9) \quad U(Z) = (1/k)Z^k,$$

where $k < 1$ to assure concavity, and $d = (1-k)$ denotes the degree of relative risk aversion. The utility of bequest function is similarly specialized as

$$(4.10) \quad V(B,t) = [n(t)/k]B^k,$$

with $n(t)$ representing the intensity of utility derived by the individual from capital to be bequeathed to heirs (see Richard (1977)). Finally, the optimal consumption and bequest choices are now taken to be conditional on a predetermined path of optimal self-protection outlays.

These simplifications permit an explicit solution for the partial differential equation (4.5). Consider the following indirect expected utility function

$$(4.11) \quad J_1(A_1(t),t;\alpha) = [a(t)/k][A_1(t) + L(t)]^k,$$

as a solution candidate. Using the optimality conditions for consumption and bequest under fair

insurance, (4.7) and (4.8), the utility functions specified in equations (4.9) and (4.10), and the boundary conditions associated with equation (4.5), by which $a(D) = n(D)$, one can verify that (4.11) does indeed provide a unique solution for the optimal value function, with the marginal expected utility of a unit of wealth while one is alive, $a(t)$, given by

$$(4.12) \quad [a(t)]^{(1/d)} = \exp\left\{-\int_t^D x(u)du\right\} [n(D)]^{(1/d)} + \int_t^D y(u) \exp\left\{-\int_t^u x(s)ds\right\} du,$$

where $x(u) \equiv f(u) + [(\rho-rk)/(1-k)]$, and $y(u) \equiv 1+f(u)[n(u)]^{(1/d)}$.

Equations (4.11) and (4.12) provide the explicit solution for the indirect expected utility of the remaining life span we have sought. It thus becomes possible to derive explicit solutions for the optimal consumption, bequest, and the value-of-life-saving functions as well. The latter variable is the shadow price of a marginal reduction in mortality risk, v_1^* , as defined in equation (4.6):

$$v_1^*(t) = [\partial J_{1i}(A_1(t), t; \alpha) / \partial f(t)] / J_{1A}(A_1(t), t; \alpha), \text{ or}$$

$$(4.6a) \quad v_1^*(t) = [J_1(A_1(t), t; \alpha) - V(B^*(t), t)] / J_{1A}(A_1(t), t; \alpha) + B^*(t) - A_1(t).$$

Equation (4.6a) summarizes, in value terms, the three basic outcomes of a marginal reduction in mortality risk in the presence of complete and actuarially fair insurance markets: (1) an increase in personal welfare by the extent of the difference between the utility from living and dying,⁹ $(1/J_{1A})[J_1(A_1) - V(B^*)]$; (2) a reduction in the total return on investment in actuarial notes, if the individual is a net saver, due to the fall in the risk premium paid on such notes, $-A_1$; and (3) a reduction in the opportunity cost of bequest (whether in the form of life insurance or regular saving) due to the fall in the insurance premium $(+B)$. A more useful summary of the net effect of these outcomes is obtained by substituting equations (4.8) through (4.11) directly in equation (4.6a) to produce a closed-form solution for the value of life saving function we have sought:

$$(4.13) \quad v_1^*(t) = z(t)[A_1(t) + L(t)] - A_1(t)$$

where $z(t) = [n(t)/a(t)]^{(1/d)} + (1/k)\{1 - [n(t)/a(t)]^{(1/d)}\}$.

Equation (4.13) represents an explicit solution for the private value of life saving as a function of personal and market-dictated parameters. This solution takes as given optimal self-protection outlays, although equation (4.6) implies that these outlays, in turn, are determined by the optimal value of life in equation (4.13). It is possible, however, to relax the assumption that optimal self-protection outlays are given, and proceed with a simultaneous solution of equations (4.6) and (4.13) through simulation analyses. Ehrlich and Yin (1999) pursue such simulations, which confirm the validity of the closed-form solutions for the value of life saving in equation (4.13), as well as for all the other control variables of the model summarized below.¹⁰

The explicit solutions for optimal consumption and bequest are similarly obtained by exploiting equations (4.7), (4.8), and (4.11) as follows:

$$(4.7a) Z^*(t) = [1/a(t)]^{(1/d)} [A_1(t) + L(t)], \text{ and}$$

$$(4.8a) B^*(t) = [n(t)/a(t)]^{(1/d)} [A_1(t) + L(t)].$$

These solutions specify both consumption and bequest to be strictly proportional to current expected wealth at any point in time. Furthermore, by inserting the explicit solution for bequest in equation (4.8a), the closed-form solution for the optimal value of life saving can be rewritten as

$$(4.13a) v_1^*(t) = (1/k)L(t) + [(1-k)/k][A_1(t) - B^*(t)].$$

The term $A_1(t) - B^*(t) \equiv Q(t)$ represents the amount of actuarial notes owned by the individual. This alternative measure of the value of life saving has the further merit of being estimable empirically using available data on individual human wealth, the net amount of actuarial notes, and independent assessments of the coefficient of relative risk aversion $d=(1-k)$.

4.C. Behavioral Propositions

Certain restrictions apply to the parameters of the demand and value functions just introduced. As

long as life insurance and annuities are available at actuarially fair terms, the magnitude of $z(t)$ in equation (4.13) will necessarily exceed unity in all periods of life. Indeed, by combining the implicit stability and optimality conditions, which require that the utility from living exceed that from dying (see footnote 9) and that the marginal utilities of consumption and bequest be equal at any period of life, it is easy to show that ¹¹

$$(4.14) \quad B^* \leq A_1 + L \quad \text{and} \quad (n/a) \leq 1 \quad \text{if} \quad 0 \leq k < 1 \quad ;$$

$$B^* > A_1 + L \quad \text{and} \quad (n/a) > 1 \quad \text{if} \quad k < 0.$$

But the wealth constraint (4.3) restricts $Z^*(t)$ and $B^*(t)$ in equations (4.7a) and (4.8a) to be strictly less than $A_1(t) + L(t)$ in all periods of life. Therefore only a value of $0 \leq k < 1$ is permissible under fair insurance.

Put differently, a stable solution under actuarially fair market insurance restricts both the total and the marginal utility from a dollar of own wealth not to fall short of that from bequeathed wealth, or $[a(t)/k] \geq [n(t)/k]$ and $a(t) \geq n(t)$, at all time periods. ¹² The first restriction alone guarantees that $z(t) \geq 1$, i.e., that the value of life saving, $v_1^*(t)$, remain non-negative in all periods of life, regardless of the magnitude of human wealth, $L(t)$. A negative value of life would be inconsistent with a stable equilibrium because it would generate an incentive to engage in "negative self-protection", i.e., precipitate an increase in the risk of mortality, which also rules out as optimal any positive expenditures on self-protection in earlier periods of life.

There is a clear link, according to equation (4.13), between the relative bequest preference and the value of life saving. In the rare case where a dollar bequeathed yields the same utility as a dollar enjoyed while alive, or $[n(t)/k] = [a(t)/k]$ (the "infinitely lived household" case), optimal bequest will exhaust the individual's wealth constraint, and the value of life will just equal the discounted value of expected future labor income, $L(t)$, which had served as a conventional measure of value of life in the

early literature on human capital (see Dublin and Lotka (1946)). The intuitive reason is that in this case a person derives equal satisfaction from a given amount of financial wealth, whether directly or vicariously through survivors, but forgoes the benefits of expected future labor income in the event of death.

The analysis is consistent with Conley's (1976) proposition that the value of life saving **exceeds** one's expected human wealth. It shows this to be the case under actuarially fair insurance even when individuals have a strong bequest motive, provided they are net savers, or $A_1(t) > 0$ (but see fn.12). If a person is a net borrower ($A_1(t) < 0$), however, the optimized value of life is seen to approach one's expected net labor income (from above) as borrowing approaches its upper bound $A_1(t) = -L(t)$, regardless of bequest preferences. In contrast, the analysis shows that in periods where the relative bequest preference $n(t)/a(t)$ is negligible, the value of life may exceed the sum of **both** human and non-human wealth if the degree of relative risk aversion in equation (4.9) is sufficiently high, or $(1/2) \leq d < 1$. Formally

$$\begin{aligned}
 (4.15) \quad v_1(t) &= L(t), \quad \text{if } n(t)/a(t) = 1 \\
 &= z(t)[A_1(t)+L(t)]-A_1 > L(t), \quad \text{if } n(t)/a(t)<1 \text{ and } A_1(t) \geq 0 \\
 &\approx L(t), \quad \text{if } n(t)/a(t)<1 \text{ and } A_1(t) \approx -L(t) \\
 &= (1/k)[L(u)+(1-k)A_1(u)], \quad \text{if } n(u)/a(u) \approx 0.
 \end{aligned}$$

The private value-of-life measure *a la* Schelling is thus linked parametrically with more traditional measures based on the assessment of human and non-human wealth. The specific link is shown to depend largely, however, on relative bequest preferences, the availability and actuarial fairness of the insurance terms, and the status of the individual as a net lender or borrower.

The value-of-life-saving measure given in equation (4.13) is expected to vary systematically over the

life cycle as a function of personal characteristics and market opportunities.¹³ In general, persons with relatively high bequest preferences (n/a) will have a lower value of $z(t)$, and hence of life saving.

Assuming that $z(t)$ does not vary considerably with age, equation (4.13) also implies that one's value of life would generally be rising with (especially young) age, as long as the accumulation of non-human assets (or the settling of past debts) rises sufficiently faster than the eventual decline in the value of human wealth due to the contracting life expectancy, but it must ultimately fall at a sufficiently old age. Indeed, the boundary conditions in equation (4.5) imply that as one reaches the final phase of the planning (but not necessarily actual) horizon, $n(D) = a(D)$ in equations (4.10) and (4.11), and thus the 'value of life' becomes nil ($v^*(D) = L(D) = 0$).

As in the analysis of section 3, a greater endowment of **non-human** capital is shown to increase the value of life saving unambiguously (as long as $(n/a) < 1$) because it generates a sufficiently large increase in the value of the remaining life span to more than offset the fall in annuity income (if $A_1(t) > 0$), or the reduction in debt service gains (if $A_1(t) < 0$), because of the decline in the risk-premium yield on actuarial notes. An increase in **human** capital (L), in contrast, is expected to cause an even **larger** increase in one's valuation of life, technically because the increase in wealth and utility it generates is not offset by any change in the capital loss component of equation (4.13).

The intuition behind this result is straightforward: Non-human capital survives its owner, while human capital does not. Given the opportunity to purchase actuarial notes, one can surrender one's accumulated non-human assets to an insurance company in return for an added annuity premium, proportional to one's conditional risk of mortality, which reflects the undisturbed market value of the surrendered assets in the event of the original owner's death. No such option is available to the owner of human capital since the market value of the latter is conditional on the future earnings it generates. The only way to buy an actuarial note with human capital is by selling an actuarial note drawn against it.

This is why a reduction in mortality causes, in part, a capital loss to the owner of a non-human asset, but it generates a gain to the owner of a human asset. Put differently, the only way to realize the market value of the latter asset is by protecting longevity. The existence of a market for actuarial notes, which establishes the (uncertain) market value of human capital, also establishes its greater importance relative to non-human wealth as a determinant of the private value of life protection. This relative importance of human wealth exists regardless of any bequest motive, but it becomes larger the higher the bequest preferences.¹⁴

A larger health endowment, as indicated by a lower biological risk of mortality $j(t)$, generally has an ambiguous effect on the value of life saving (see section 2). It depends, in part, on whether the level of $j(t)$ per se has any systematic effect on the productivity of self-protection efforts. The value of life saving is directly affected, however, by the personal level of human wealth, $L(t)$, which is an unambiguously decreasing function of mortality risks. By this analysis, the value of life saving, and thus expenditures on life protection, is likely to be higher the lower the mortality odds. Equation (4.13) also indicates that a higher degree of **relative risk aversion** $d = (1-k)$ unambiguously raises the value of life saving.

The incentives to buy market insurance and to provide self-protection are interrelated by their responsiveness to specific parameters of the model. For example, equations (4.8a) and (4.13) indicate that an exogenous increase in wealth raises the optimal value of bequest and the value of life saving, and thus, generally, the demand for both life insurance and self-protection (see equation (4.6)). An increase in relative bequest preferences, in contrast, increases the incentive to provide life insurance, but decreases the incentive for self-protection. Thus, life insurance and self-protection are likely to be **complements** with respect to a change in wealth (regardless of its source) and **substitutes** with regard to a change in bequest preference.

5. The Role of Annuities and Life Insurance

A frequent concern in the economic literature on insurance is the potentially adverse effect of insurance on self-protection, commonly termed "moral hazard". Do actuarially fair markets for annuities and life insurance create such a hazard in connection with life protection? Following the approach pursued in EB (1972), the question could be answered by assessing how the introduction of complete insurance markets affects the shadow price of the conditional mortality risk, or value of life saving, in its **general** form, as specified in equation (4.6).

To make things comparable, assume that individuals have a bequest motive, and that all the parameters of the model are invariant to the opening of insurance markets in period 0, including the initial assets' level (i.e., $A_{10} = A_0$). In the absence of any insurance, equation (4.6) changes to

$v_0^* = (1/J_A)[J - V(A_0)]$, while under actuarially fair insurance markets it becomes

$v_1^* = (1/J_{1A})[J_1 - V(B^*)] + B^*(t) - A_0(t)$. The difference between these two value-of-life-saving levels can be assessed as follows:

$$(5.1) \quad v_1^* - v_0^* = D_1 + D_2 ,$$

where $D_1 \equiv (J_1/J_{1A}) - (J/J_A)$, and $D_2 \equiv [(V(A_0)/J_A) - A_0] - [(V(B^*)/V'(B^*)) - B^*]$ after replacing J_{1A} with $V'(B^*)$ in the expression for v_1 , based on the optimality condition for bequest in equation (4.8).

The term D_1 in equation (5.1) is expected to be positive in sign, essentially because the opportunity to trade in the market for actuarial notes enables individuals to choose optimal paths of future bequest and consumption independently of each other. Moreover, this market expands the effective wealth constraint at any given period from $A_0(t) \geq 0$ (see section 2) to $A_0(t) + L(t) \geq 0$. The resulting added efficiency in inter-temporal resource allocation implies that $D_1 \geq 0$.

The term D_2 summarizes two additional outcomes: 1. A change in actual bequest level, because of the opportunity to select an optimal amount of this commodity; 2. A reduction in the opportunity cost of

bequest (the life insurance premium) but also in the extra return the insurer can offer on annuities, because of the fall in mortality risk. The strict concavity of the utility function implies that D_2 is positive if $A_0 > B^*$ and negative if $A_0 < B^*$.¹⁵

A fall in actual bequest relative to accidental bequest -- i.e., $A_0 > B^*$, -- occurs if a person takes advantage of the opening of insurance markets to purchase annuities as well as, possibly, life insurance as a substitute for regular savings. This outcome is possible since in the pre-insurance period, accumulated savings serves as a means of self-insuring both future consumption needs and bequest. The level of accidental bequest implied by savings may exceed the desired level of bequest, given the ability to set it independently. The purchase of annuities eliminates the inefficiency implicit in such accidental bequest, and in this case, as is formally proven (see fn. 15), there is an unambiguous increase in the value of life saving. The intuitive reason is that if the optimally chosen bequest is smaller than the previously uncontrolled (accidental) bequest, the disparity between the utility levels in the states "alive" and "dead" becomes larger, and this increases the private value of life protection.

It is also possible, however, that for people with a high bequest preference, the previously accumulated amount of regular savings falls short of the desired level of bequest they can now secure by borrowing against their future earnings. The inequality $A_0 < B^*$, signaling an increase in the optimal level of bequest, is made possible through the selling of actuarial notes; i.e., if individuals purchase only life insurance, but no annuities.¹⁶ Utilizing the insurance system exclusively to promote the welfare of future generations is expected to lower the value of one's own life saving, as our analysis of the distinct role of bequest preferences has shown.

What if individuals have no bequest motive before or after the introduction of fair insurance? By the preceding analysis, $v_1 > v_0$, when $A_0 > B^* > 0$. But by equation (5.1) the value of life saving is a monotonically decreasing function of relative bequest preferences, so that when the latter approaches

zero, v_1 peaks! By transitivity we conclude, therefore, that persons with no bequest motive stand to benefit the most from the opening of insurance markets by specializing in annuity purchasing.

The interesting conclusion of this analysis is that the emergence of actuarially fair insurance markets will necessarily raise the value of life saving, and hence the demand for life protection, as long as individuals take advantage of these markets to purchase annuities as well as life insurance. This generalizes the analysis of "moral hazard" in EB (1972), where the introduction of fair insurance had a more ambiguous effect on self-protection.

Furthermore, these results are independent of any specific utility function or age-earning profile, and they may apply even under actuarially unfair insurance terms, provided that insurance premiums are responsive to individuals' true odds of mortality.¹⁷ If the price of insurance is not responsive to such odds, inefficient self-protection may be an inevitable outcome of insurance (cf. EB, op. cit.), but its direction in this case will be ambiguous. Whereas the purchase of life insurance may depress investment in life protection, that of annuities creates an incentive to increase it. Insurance companies may seek to minimize such inefficiencies by offering customers a package of life insurance and lifetime annuities or pension funds.

Insurance, Life Protection, and Life Saving: Concluding Remarks

Many of this paper's inferences concerning the determinants of health and life expectancy are qualitatively similar to those derived in earlier analyses where health investment outcomes, including longevity, were treated as known with certainty.¹⁸ By incorporating uncertainty, self-protection, and alternative insurance options in the formal analysis, however, the present model offers a number of additional insights. These include the roles of bequest preferences, attitudes toward risk, and the availability and actuarial fairness of markets for annuities and life insurance.

The existence of complete insurance markets affects not just the level of demand for protection, but also the relative importance of some of its previously recognized determinants. Under actuarially fair insurance, an increase in human wealth is expected to generate a greater increase in the demand for life protection relative to non-human wealth, even if human capital were not serving as an efficiency parameter in health production. This result is not necessarily forthcoming in a world without insurance: In that world, the impact of human capital would not be directly comparable to that of other forms of wealth, since opportunities to borrow against future earnings would then be constrained by the uncertainty regarding life's length. The relative importance of human capital in determining longevity would be further exposed if it were recognized as an "engine of growth" in the context of endogenous growth. As such, human capital (and related medical technology advances) can be expected to account for much of the continuing **growth** in per-capita income and wealth, as well as in life expectancy, in most countries over the last two centuries. It also provides an explanation for the significant and persistent **inequality** in life expectancy across developing and developed countries.

The existence of insurance increases the likelihood that larger health endowments (lower mortality risks) raise the demand for life protection. Although Table 2 indicates that their effect may be ambiguous in the absence of insurance, even when account is taken of the impact of lower morbidity risks on healthy time, equation (4.13) indicates that lower mortality risks increase the discounted value of potential future earnings, or human wealth. The healthy are likely to become even healthier by this analysis, which may explain why females' life expectancy has gone up more than males' over the 20th century, or why older age groups have been experiencing relatively larger percentage increases in life expectancy over time. Biological evidence indicates that females' health endowments are indeed superior to males' (see, e.g., Guido, 1965), and people reaching older age are naturally self-selected for survival.

It is not easy to identify empirical counterparts for relative bequest preferences, but one may conjecture, following Becker et al. (1990), that the intensity of the current generation's bequest motive (n/a in equation (4.13)) is directly related to fertility, or average family size. Since the analysis of section 4 shows that the private value of life protection is inversely related to one's relative bequest preferences, it follows that a reduction in average family size will be associated with a higher demand for life expectancy. One would expect an inverse relationship between falling fertility rates and rising survival odds over time, which is what the empirical evidence shows for most developed countries in recent decades. And to the extent that the declining fertility trends themselves are an endogenous outcome of human capital accumulation (cf. Ehrlich and Lui, 1991), the differential impact of human capital on longevity over and above other sources of wealth (not controlling for family size) would be even more pronounced. A higher degree of **relative risk aversion** is also shown to raise unambiguously the demand for life protection, which indicates a potential association between less risky portfolio compositions and longevity odds.

A unique implication of the model is that the existence of a complete, actuarially fair market for actuarial notes increases the demand for life protection as long as individuals purchase both annuities and life insurance. Moreover, the association between life insurance and life protection is expected to be negative, while the converse holds for annuity insurance, acquired through defined-contributions, private pension plans, essentially because a larger preference for bequest decreases the optimal value of life saving.

Social insurance programs, such as social security, play a different role. To the extent that social security were to be structured as a fully funded, defined-contributions system, it would simply simulate a private annuities market. Under a "pay-as-you-go" system, in contrast, the effect is ambiguous. On the one hand, an unfunded, defined-benefits social security system may affect adversely either fertility,

human capital accumulation, or savings (and possibly all three; see Ehrlich and Lui, 1998), which would lead to conflicting effects on the demand for life protection. On the other hand, such a system also generates a reverse "moral hazard" effect since, by design, both the insurance premiums and the retirement benefits are not adjusted in proportion to their survival risks. The latter feature of social security creates an incentive to increase private outlays on health and life protection, to enjoy the guaranteed annuity benefits over a longer horizon.

The model's solutions for the optimal values of all the control variables, including consumption, bequest, and self-protection, could be implemented empirically or through calibrated simulations. These would reveal the quantitative importance of individual self-protection in explaining both the trend and inequality in mortality risks and life expectancies across different population and age groups. Moreover, the analysis in section 4 indicates the potential of wide variability in value-of-life-saving estimates by age, occupation, education, sex, wealth, and family status. It thus provides a theoretical explanation for the wide variability in empirical estimates of the value of life saving, based on the willingness-to-pay approach, which range between substantially less than \$1 million to substantially over \$10 million (see Viscusi, 1993). Providing additional insights into the quantitative importance of health and life protection in explaining the trend and diversity in life expectancies and value-of-life-saving measures may be an important challenge for future work.

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TABLE 1

Key Variables Names

\tilde{D}, D, T^*	The uncertain, maximal, and expected length of life
$j(t), f(t)$	The exogenous and endogenous conditional probability of mortality in t given survival to t
$h(t)$	Amount or fraction of healthy time in t
$Z(t)$	Flow of consumption services
$B(t), V(t)$	Total bequest and its utility, respectively
$U(t)$	Instantaneous utility of "quality of life"
$I(t)$	Flow of self-protective activity
$M(t)$	Flow of medical care inputs in the production of $I(t)$
$m(t)$	Personal time inputs in the production of $I(t)$
Ω	Total time constraint in t exhausting healthy time and sick time.
$E'(t)$	Stock of education
$A(t)$	Stock of nonhuman assets at t
$L(t)$	The discounted value of potential future labor income net of self-protection outlays
$w^0(t)$	Market wage rate; $w^0(t) = wE(t)$
$P(t)$	Unit price of medical care services, $M(t)$
$c(t)$	One-unit (shadow) price of $I(T)$
r, ρ	Market and psychological discount rates
β	A shift parameter representing effort-saving technological innovations in equation (1.2)
d	A relative risk aversion parameter, $d = 1-k$
$n(t)$	Utility of a one unit of capital to be bequeathed at t

$J(t), J_A(t)$ The maximized expected utility and the expected marginal utility of wealth over the remaining life span at t

$v(t), v_1(t)$ Alternative definitions of value of life saving at t

TABLE 2: COMPARATIVE- STATICS PREDICTIONS

Parameter (α) Variable	A(t)	P	E'	w ^(a)	j	r ^(d)	ρ	β ^(e)
$I_\alpha(t) \equiv \partial I^*(t) \partial \alpha$	>0	<0	>0	>0	?	>0	<0	>0
$Z_\alpha(t)$	>0	<0	>0	>0	<0	>0	<0	>0
$v_\alpha(t)$	>0	≥ 0 ^(b) <	≥ 0 ^(b) <	>0	?	>0	<0	>0
$T(t)$ ^(c)	>0	<0	>0	>0	?	>0	<0	>0

(a) Provided that $c_w \cong 0$

(b) Since $v^*(t) = 2cI^*$ the sign of $v_\alpha(t)$ depends on whether the percentage change in $I^*(t)$ is greater than the opposite percentage change in $c(t)$.

(c) $T^*(t)$ denotes life expectancies (see text).

(d) Assuming that $\text{sgn}(-J_{AR}) = \text{sgn}(J_r) > 0$ (see text); $J_{AR} \equiv \partial^2 J / \partial A \partial r$.

(e) This parameter represents effort saving technological innovations which augment the effective amount of productive time available at any given state of health. Formally, $h=h(f; \beta)$ with $h_\beta > 0$.

FOOTNOTES

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¹ The strict linearity of $f(t)$ with respect to $I(t)$, given $j(t)$, is assumed for computational convenience, since the production function of $I(t)$ itself is modeled to exhibit diminishing returns to scale. Also, $I(t)$ is taken to be technically independent of $j(t)$ or t in the formal analysis because the direction of any such dependency would be a matter of speculation. The formulation also abstracts from the possibility that current expenditures on self-protection could have lingering effects on conditional mortality risks in future periods (but see footnote 18). These assumptions simplify the derivation of a dynamic path for $I^*(t)$ in the following sections. But they can be relaxed analytically or in simulations without loss of generality (see, e.g., footnote 3).

² Unlike the analysis in Ehrlich and Chuma (1990), where a restriction on a minimum level of health required for living (H_{\min}) was imposed to dictate a finite value for the **certain life span**, in this analysis the **planning horizon** is not restricted to require a minimum level of survival risk. Thus, life may be possible as long as $p(a, b) = \varepsilon > 0$. The simplest assumption may be that $D = \infty$. In practice, however, D can be set at a level where the optimal value function $J(D)$ in equation (2.2) (the expected utility from life) becomes virtually invariant to the length of the planning horizon. Interestingly, simulation analyses by Ehrlich and Yin (1998) show the age level of D to be 105 years, based on recent data concerning mortality risks for the total US population

³ More generally, if $f(t)$ in equation (1.1) were written as

$f(I(t)) = K(t)G(I(t))$, with $G'(I) < 0$ and $G''(I) > 0$, then the marginal cost of reducing f in equation (2.5) would be given by $[-2cI]/[KG'(I)]$.

⁴ The effect of a change in any exogenous parameter, α , on the optimal values of Z and I can easily be determined using equations (2.4) and (2.5). For example, for $\alpha = A$,

$\partial I^*/\partial A = [1/(\partial K/\partial I)]\{(J/J_A)[(J_A/J) - (J_{AA}/J_A)] + (U_h h' J_{AA}/J_A^2)\} = (+)/(+) > 0$, where

$K \equiv (\partial J/\partial I) (1/J_A) = 2cI - (J/J_A) + [w + (U_h/J_A)]h'$, and therefore $\text{sgn}(-\partial K/\partial I) = \text{sgn}(-\partial J/\partial I) < 0$ by the second order condition for I^* to maximize equation (2.3). Similarly, $\partial Z^*/\partial A = (J_{AA}/U_{ZZ}) > 0$, provided that $\text{sgn}(-J_{AA}) = \text{sgn} J_A > 0$.

⁵ In this analysis mortality and morbidity risks are perceived to be technically independent of consumption. As has been argued in the literature, since some consumption activities can be hazardous to one's health, higher wealth may theoretically result in higher risks to health and life. The argument ignores, however, the possibility of controlling hazardous consumption characteristics by choosing alternative commodity brands. To the extent that hazardous and safe consumption characteristics can be unbundled and controlled through branding or safety measures, higher wealth would be expected to increase both, as well as life protection. For example, higher wealth is likely to increase the demand for both luxury and safety components of sport utility vehicles, such as leather seats, airbags, and wide tires, let alone safe driving.

⁶ The time path of self-protection outlays under annuities remains formally the same as without annuities (equation (2.6)), except that it reflects the effects of the higher annuity return $r+f(t)$ on the value of reducing mortality risk. The first two terms in the numerator of the R.H.S. of (2.6) change to $d[(J_I/J_{IA}) - A_I]/dt + (U_h/J_{IA})(-h')(r-p)$. The qualitative effect of this change by the preceding analysis is

to make health spending rise more pronouncedly with age due to the increased incentive to defer spending to the future.

⁷ See, in particular, the studies by Thaler and Rosen (1975), Conley (1976), Viscusi (1978, 1978a), Arthur (1981), Shepard and Zeckhauser (1984), and the articles in Jones Lee (1982). Some of these studies have discussed the effect of specific variables on the value of life saving, and have attempted to link the latter with the discounted value of labor income. But none has developed a comprehensive "indirect" value function, which accounts specifically for the role of bequest preferences, borrowing and saving decisions, age, components of wealth, and insurance markets in affecting the value of a marginal reduction in mortality risk.

⁸ That is, $L(t) = \int_t^D \exp[-r(u-t)-m(t,u)][wh(u)-cI^2(u)]du$. This expression represents the discounted value of net potential future labor income -- i.e., the discounted gross value of all healthy time net of life protection costs, including the opportunity costs of time spent at health preservation.

⁹ The solution must also satisfy the stability condition $J_1(A(t), t; \alpha) > V(B^*(t), t)$, $t < D$, by which the utility from living must exceed that from dying during all periods of life. Of course, by the boundary conditions, $v_1^* = 0$ at $t = D$, since $J_1(D)=V(B)$, and $B = A_1(D)$.

¹⁰ An elaborate calibrated simulation analysis is pursued in Ehrlich and Yin (1999) whereby self-protection, $\bar{I}^*(t)$, and the corresponding path of mortality risks and current human wealth are simultaneously determined with $v_1^*(t)$, $Z^*(t)$, and $B^*(t)$. These simulations produce unique and stable dynamic solutions for equations (4.13), (4.7a), and (4.8a), as well as for $\bar{I}^*(t)$, thus proving the validity of the closed-form solutions summarized by these equations.

¹¹ The conditions $(n/k)B^k < (a/k)[A_1+L]^k$ and $nB^{k-1} = a[A_1+L]^{k-1}$ together imply that $(B/k) < (A_1+L)/k$, so that $B > (<) (A_1+L)$ if $k < (>) 0$.

¹² If an identical loading factor $q > 0$ were applied to both lending and borrowing transactions in actuarial notes, raising the risk premium for both from f to $(1 + q)f$ (see equation 4.5), equation (4.12)

would remain valid except that the functions $x(u)$ and $y(u)$ would become

$x_1(u) \equiv (1/d) \{f(u)[1-(1 + q)f] + \rho - rk\}$ and $y_1(u) \equiv 1 + (1 + q)^{-(k/d)} f(u)n(u)^{(1/d)}$, respectively, and

equation (4.13) would become (4.13a) $v_2^*(t) = z_1(t)[A_1(t)+L(t)] - (1 + q)A_1(t)$,

where $z_1(t) \equiv [1+q]^{-(k/d)}[n(t)/a(t)]^{(1/d)} + (1/k)\{1-[1+q]^{-(k/d)}[n(t)/a(t)]^{(1/d)}\}$.

By the reasoning underlying the derivation of equation (4.15), one cannot rule out in this case the possibility that $(n/k) > (a/k)$, provided that $[n(t)/a(t)] \leq (1+q)$, which guarantees the equilibrium condition that $B^*(t) = \{[n(t)/a(t)]/(1+q)\}^{(1/d)}[A_1(t)+L(t)] < A_1(t)+L(t)$. In this case $z_1(t)$ would be less than unity and $v_2(t)$ could be **less** than $L(t)$ if $A_1(t) > 0$ (see equation (4.13a) above). This result may be of limited empirical value, however, since the existence of transaction costs in insurance suggests that loading factors of **opposite** signs should be applied to borrowing and lending transactions. No general solutions may be obtained under such insurance terms.

¹³ Although the comparative dynamic predictions below are obtained directly from equation (4.13), which takes optimal self-protection expenditures as given, they have been confirmed through simulation analyses that allow a simultaneous determination of self-protection and all other control and co-state variables (see footnote 10).

¹⁴ If an asset is transferable, its owner suffers a smaller utility loss from the prospect of death the higher is his bequest preference. Indeed, the differentially greater impact of human relative to non-human wealth on the value of life saving, as measured by $z(t)/[z(t)-1]$ in equation (4.13), becomes even larger the higher the person's relative bequest preferences, $n(t)/a(t)$.

¹⁵ To prove this proposition recall the familiar property of a strictly concave function that $\{[V(x)/V'(x)]-x\} > \{[V(y)/V'(y)]-y\}$ whenever $x > y$. Since $V(x)$ is unique up to a linear transformation,

one may set $V(A_0)=0$ without loss of generality. Consequently, one may substitute in equation (5.1) $V(A_0)/V'(A_0) = 0$ for $V(A_0)/J_A$, and the concavity property of V just cited implies that $D_2 > (<) 0$ as $A_0 > (<) B^*$.

¹⁶ Defining the stock values of ordinary saving notes, actuarial notes, and life insurance by S , Q , and N respectively, one can write $B^* = S+N$ and $A_{10} = Q+B^*$. In the present case where $A_{10} = A_0$ and $B^* > 0$ by assumption, if $A_0 - B^* > 0$, then $Q > 0$ and $S+N > 0$. Both annuities and life insurance may be purchased in this case. But if $A_0 - B^* < 0$ then $Q < 0$ and $S+N > 0$: To effect an increase in bequest above initial assets a person must sell an actuarial note to buy more regular saving notes, i.e., purchase only a conventional life insurance policy but no annuities.

¹⁷ Assuming, as in section 3, that opposite loading terms of equal magnitude q are applied to annuities and life insurance premiums, but that premiums still reflect individuals' true probabilities of mortality (i.e., they respond to individual self-protection), equation (4.6) becomes

$$v_1 = (1/J_{1A})[J_1 - V(B^*)] + (1+q)B^* - (1-q)A_0 \text{ and equation (5.1) becomes}$$

$v_1 - v_0 = D_1 + D_2 + q(A_0 + B^*)$, where the last term represents extra savings from a reduction in mortality risk due to the presence of insurance "loading" charges. Thus, as long as $A_0 > B^*$, the value of life protection increases with unfair insurance relative to its magnitude when **no** insurance is available. It is less clear, however, whether unfair insurance reduces the value of life saving below its magnitude under **fair** insurance. Actuarially unfair insurance terms lead to a reduction in either optimal bequest or annuities (which would have opposing effects on the value of self-protection) and it lowers the efficiency gain from insurance, relative to actuarially fair terms.

¹⁸ See, e.g., Grossman, 1972, and Ehrlich and Chuma, 1990. This is despite the fact that self-protection is treated here as contributing to the flow, rather than the stock of health, unlike the referenced papers. If self-protection $I(t)$ had a lingering effect (subject to depreciation) on future

conditional mortality or morbidity risks, the main change in the analysis would be the shape of the optimal paths of $I^*(t)$ and $v^*(t)$ over the life cycle. The marginal benefits from self-protection would be higher at any period of life. But because the impact of self-protection depreciates over time, this creates an incentive to delay spending to later phases of the planning horizon, thus increasing the value of life saving (see Ehrlich and Chuma, op. cit).