University at Buffalo
GIFTED MATH PROGRAM

Course III Curriculum

Textbooks: Comprehensive School Mathematics Program, *Elements of Mathematics*, Book 0, Chapter 14; *Mira Math Activities for High School*, a Mira Math Co. Publication; *Geometry: Seeing, Doing, Understanding* (3rd edition) by Harold Jacobs; and *Transformational Geometry* by Richard Brown

Unit 1: GEOMETRY: ***Geometry: Seeing, Doing, Understanding***

- CHAPTER 1 – AN INTRODUCTION TO GEOMETRY: Angles in Measuring the Earth, Pyramid Architecture, Telling Time with Shadows, Ancient Formulae
- CHAPTER 2 – THE NATURE OF DEDUCTIVE REASONING: Conditional Statements; Definitions; Direct Proof; Indirect Proof; A Deductive System; Some Famous Theorems of Geometry
- CHAPTER 3 – LINES AND ANGLES: Number Operations and Equality; The Ruler and Distance; The Protractor and Angle Measure; Bisection; Complementary and Supplementary Angles; Linear Pairs and Vertical Angles; Perpendicular and Parallel Lines
- CHAPTER 4 – CONGRUENCE: Coordinates and Distance; Polygons and Congruence; Congruence Proofs; Isosceles and Equilateral Triangles
- CHAPTER 5 – INEQUALITIES: The Exterior Angle Theorem; Triangle Side and Angle Inequalities; The Triangle Inequality Theorem
- CHAPTER 6 – PARALLEL LINES: Line Symmetry; Proving Lines Parallel; The Parallel Postulate; Parallel Lines and Angles
- CHAPTER 7 – QUADRILATERALS: Quadrilaterals; Parallelograms and Point Symmetry; Rectangles, Rhombuses, and Squares; Trapezoids; The Midsegment Theorem
- CHAPTER 8 – TRANSFORMATIONS: Transformations; Reflections; Isometries and Congruence; Transformations and Symmetry
- CHAPTER 9 – AREA: Area; Squares and Rectangles; Triangles; Parallelograms and Trapezoids; The Pythagorean Theorem
- CHAPTER 10 – SIMILARITY: Ratio and Proportion; Similar Figures; Perimeters and Areas of Similar Figures
- CHAPTER 11 – THE RIGHT TRIANGLE: Proportions in a Right Triangle; Isosceles and 30^{O}-60^{O} Right Triangles; The Tangent Ratio The Sine and Cosine Ratios; Slope
- CHAPTER 12 – CIRCLES: Circles, Radii, and Chords; Tangents; Central Angles and Arcs; Inscribed Angles; Secant Angles; Tangent Segments and Intersecting Chords; The Laws of Sine, Cosine and Tangent
- CHAPTER 13 – THE CONCURRENCE THEOREMS: Triangles and Circles; Cyclic Quadrilaterals; Incircles; The Centroid of a Triangle; Ceva's Theorem; Napoleon's Discovery and Other Surprises
- CHAPTER 14 – REGULAR POLYGONS AND THE CIRCLE: Regular Polygons; The Perimeter and Area of a Regular Polygon; From Polygons to Pi; The Area of a Circle; Sectors and Arcs
- CHAPTER 15 – GEOMETRIC SOLIDS: Lines and Planes in Space; Rectangular Solids; Prisms; Pyramids; Cylinders and Cones; Spheres; Similar Solids; The Regular Polyhedra
- CHAPTER 16 – NON-EUCLIDEAN GEOMETRIES: Geometry on a Sphere; The Saccheri Quadrilateral; The Geometries of Lobachevsky and Riemann; The Triangle Angle Sum Theorem

Unit 2: CONSTRUCTIONS

- SECTION 1 – FROM *MIRA MATH ACTIVITIES FOR HIGH SCHOOL:* Construction and Properties of Perpendicular Lines; The Meaning of Reflection; Construction and Properties of Geometric Figures; Symmetry; Investigation of Properties of Geometric Figures; Motion Geometry
- SECTION 2 – CONSTRUCTIONS WITH COMPASS AND STRAIGHTEDGE

Course III Curriculum
(Continued)

Unit 3: TRANSFORMATIONAL GEOMETRY

- CHAPTER 1 – THE GEOMETRY OF TRANSFORMATIONS: Maps and Mappings; One-to-One Mappings; Transformations; Mappings in Algebra; Functions; Isometries; Problems Solved by Reflections; Properties of Isometries; Rotations; Translations and Glide Reflections; Symmetry; The Fundamental Theorems of Isometries
- CHAPTER 2 – THE ALGEBRA OF TRANSFORMATIONS: The Composite (Product) of Mappings; The Algebra of Translations; The Algebra of Half-Turns; The Algebra of Rotations; Groups; Transformation Groups; Symmetry Groups

Unit 4: TOPICS IN PROBABILITY AND STATISTICS

- REVIEW PROBLEMS: Random walks
- EXPECTATION: Definition of expectation, Practical significance of expectation, Applications of expectation
- OPERATIONS RESEARCH: Inventory problems, Group testing
- PROBLEMS AND APPLICATIONS

Unit 5: EXPLORING CONIC SECTIONS (SELECT ACTIVITIES)

- CIRCLES: Construction given center and radius; Equations of circles in coordinate plane
- ELLIPSES: Construction given foci-pins and string; Concentric circles construction; Paper folding construction; Equations of ellipses in coordinate plane; Eccentricity; Area; Applications of ellipses
- PARABOLAS: Construction given focus and directrix; Sliding ruler method; Concentric circles construction; Paper folding construction; Equations of parabolas in coordinate plane; application of parabolas
- HYPERBOLAS: Construction of hyperbolas based on foci; Concentric circles construction; Eccentricity; Equation of hyperbolas in coordinate place; asymptotes; application of hyperbolas
- OPTIMIZATION: Pre-calculus solutions (using ellipses) to Burning Tent and Swimming Pool problems