

University at Buffalo
GIFTED MATH PROGRAM

Course I Curriculum

Textbooks: Comprehensive School Mathematics Program, *Elements of Mathematics*, Book 0, Chapters 1, 3, 5, 6, 7, 8, 9; and Book 1.

Unit 1: OPERATIONAL SYSTEMS

- CODES USING MODULAR ARITHMETIC: Multiplication and division modulo 29
- MODULAR ARITHMETIC: Addition, Subtraction, Multiplication
- AN APPLICATION OF MODULAR ARITHMETIC: Time zones
- EXAMPLES OF OPERATIONAL SYSTEMS: Modular systems, Maximum, Minimum, Greatest common divisor, Least common multiple, Midpointing, Triple pointing, Reflecting in a point
- SOME PROPERTIES OF OPERATIONAL SYSTEMS: Commutativity, Associativity, Neutral elements, Invertibility, Distributivity

Unit 2: SETS, SUBSETS, AND OPERATIONS WITH SETS

- SETS AND SUBSETS: Equality, Membership, Singleton sets, The empty set, Venn diagrams, Subsets, Power sets, Number of elements of a power set, Number of k -element subsets of a set with m elements (Pascal's formula)
- OPERATIONS WITH SETS: Intersection, Union, Difference, Symmetric difference

Unit 3: MAPPING

- MAPPINGS: Mappings from A to B, Mappings from A onto B, One-to-one mappings, Permutations on a set, The image of an element under a mapping (" $f(x)$ " notation), Composition of mappings, Arithmetic mappings, Circular slide rules
- MAGNIFICATION OF MAPPINGS ON LENGTHS: "Stretchers," "Shrinkers," Composites of stretchers and shrinkers, Addition of lengths, Comparison of lengths
- APPLICATIONS OF MAGNIFICATION MAPPINGS: Photography, Map making, Scale drawing, Solution of problems on weight, time, and money
- PERCENT MAPPINGS

Unit 4: THE RATIONAL NUMBERS, DECIMALS AND AN APPLICATION OF RATIONAL NUMBERS

- THE RATIONAL NUMBER LINE: Positive and negative rational numbers, Ordering rational numbers, Density of the rational numbers, Multiplication of rational numbers, Division of rational numbers, Addition of rational numbers, Distributivity of multiplication over addition, Using mappings to solve equations, Subtraction of rational numbers, Absolute value
- DECIMALS: Addition, Subtraction, Multiplication, Approximation, Positional notation
- OPERATIONS USING POSITIONAL NOTATION: Addition, Subtraction, Multiplication, Division, Approximation, Decimals and percent
- AN APPLICATION: Income tax in a hypothetical country

Unit 5: INTRODUCTORY LOGIC

- THE FORMAL LANGUAGE: Introduction, Negation, Conjunction, Disjunction, Sentences and Well formed Formulas, Truth tables, Implication, Equivalence, the Substitution Principle, the Tautology Principle
- DEMONSTRATIONS: Modus Ponens, Conjunctive Inference, Conjunctive Simplification, Contrapositive Inference, Modus Tollens, Syllogistic Inference, Inference by Cases, Commutative Property for the Biconditional, Transitive Property for the Biconditional, Modus Ponens for the Biconditional, the General Substitution Principle, Elimination of connectives, the Deduction Theorem, Object language and meta-language, the Principle of Indirect Inference
- THE PROPOSITIONAL CALCULUS: Propositional Calculus and Open Sentences, Limitations of the Propositional Calculus, Universal and Existential Quantifiers

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Unit 6: AN INTRODUCTION TO PROBABILITY

- ONE-STAGE RANDOM EXPERIMENTS: Outcomes, Outcome Sets, Relative frequencies, Events, Probability of an event
- MULTIPLE STAGE EXPERIMENTS: Tree diagrams, The product rule, Drawings with and without replacement
- COUNTING: Product rule, Counting with restrictions, Factorials, Total number of subsets of an n-element set, Arrangements
- SAMPLING: Ordered sampling with and without replacement, Unordered sampling with and without replacement
- RELIABILITY OF A SYSTEM: Series systems, Parallel systems, Majority systems, Mixed systems
- RANDOM DIGITS AND SIMULATION: Properties of random digits, Drawing samples with random digits, Simulation, Applications

Unit 7: NUMBER THEORY

- MULTIPLES AND DIVISORS OF NATURAL NUMBERS: Primes, Composites, Twin primes, Relatively prime, Unique factorization, The infinitude of primes, Sieve of Eratosthenes, Euler Phi Function
- LEAST COMMON MULTIPLES AND GREATEST COMMON DIVISORS: Division Theorem, Number Bases, Representations in alternate bases, Divisibility theorem, LCM, GCD, Linear combinations, Euclidean algorithm, Fermat's little theorem, Chinese remainder theorem
- FACTORS IN ANOTHER OPERATIONAL SYSTEM: Divisors and multiples, 1-fold and 2-fold operational systems

Unit 8: THE PYTHAGOREAN THEOREM

- SIMILARITY: Ratio, Area, Shearing, Scaling, Rational and irrational numbers
- PROOFS OF PYTHAGOREAN THEOREM: Similar triangles, Area, Algebra, Transformations
- HISTORY: The Pythagorean School, Euclid's proof, the Chinese proof, President Garfield's proof, Leonardo DaVinci's proof, Pythagorean Triples
- APPLICATIONS: Isosceles right triangle, Bisected equilateral triangle, Constructions of irrational numbers, Spirals
- CONVERSE OF PYTHAGOREAN THEOREM
- EXTENDING PYTHAGOREAN THEOREM TO THREE DIMENSIONS

Unit 9: PI

- WHAT IS THE NUMBER π ?: Ratio of circumference to diameter as a fundamental constant of nature, circumference and area of circles
- SOME USES OF π : Volume of a cylinder, Surface area of a cylinder, Role of William Jones, John Machin, Leonhard Euler, π as first letter of the Greek words for perimeter and periphery, Pappus' rule for surface area, Pappus' rule for volume of a solid, solids of rotation
- EARLY HISTORY OF π : Babylonian, Egyptian, Biblical, Greek, Chinese, Hindu approximations, infinite series and products of ratios of integers that yield π
- COMPUTATION OF π : A Discovery of Archimedes, Pi determined by slicing a circular disk, Pi determined by interior and exterior regular polygons, Johann Lambert's role showing π is irrational using continued fractions
- PI MISCELLANY: Buffon Needle Problem, Using Random Numbers to approximate π