

→ Have seen that Bounding spheres and Bounding  $\textcircled{P}$  Rectangles are most common,

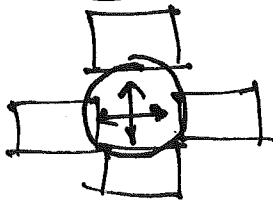
? How to mix and match?

→ Consider projections of B.S & B.R on primary planes

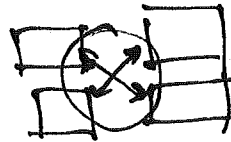
{ Unlike sweep and plane which were projections onto the axis }

→ given pairwise comparison 9 cases occur

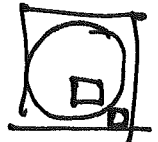
Circle  $(C, R)$       Rectangle  $(R)$



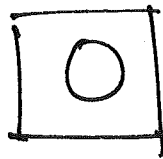
or



or



or



Algos:

→ Translate ~~C to origin~~ <sup>origin to C</sup> { R.H. Coordinate system }

→ compute  $R_{max} = R_{max} - C$ ;  $R_{min} = R_{min} - C$

IF  $(R_{max} \cdot x < 0)$  R to left of circle center

THEN IF  $(R_{max} \cdot y < 0)$  lower left

~~ELSE~~ IF  $(R_{max} \cdot y > 0)$  upper left

~~ELSE~~ To the west

(2)

→ Return condition  $(R_{max} \cdot x^2 + R_{max} \cdot y^2 < Rad^2)$

ELSE To the WEST

→ Return  $(R_{max} \cdot x) < Rad$

ELSE IF  $(R_{min} \cdot x > 0)$  to the right

{ check for lower  
or upper

→ Return  $(R_{min} \cdot x^2 + R_{min} \cdot y^2 < Rad^2)$

→ Return  $(R_{min} \cdot x < Rad)$

ELSE

check for south

check for north.

ELSE

R contains circle point.

Corollary: what is the general way of finding intersection b/n a box & sphere?  $\updownarrow$

# - Efficient Bounding Spheres

→ Given "N" points in space find bounding

→ Algo  $O(N)$

↳ Best to find max  $X^2 + Y^2$   
min  $X^2 + Y^2$

→ one pair amongst these will give greatest distance

→ fix as dia,

→  $r = \text{dia} / 2$ , center =  $\frac{\text{max} - \text{min}}{2}$

Alternate  
- why not use euclidean distance?

↳ Iterate thro' point

→ for given point

$$|F| \left( \begin{matrix} x_c^2 + y_c^2 + z_c^2 < \\ \text{radius}^2 \end{matrix} \right)$$

$$\bar{x}_c = \text{center} - x$$

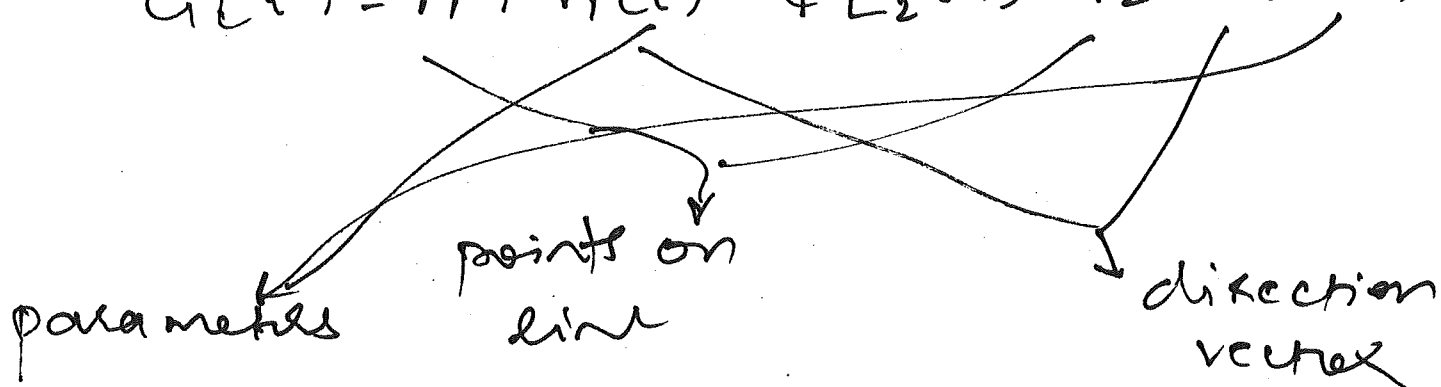
→ update sphere with new point and radius

→ worst case (Octagonal Box of points)

# Intersecting lines in 3 space

(4)

$$L_1(t) = P_1 + v_1(t) \quad \& \quad L_2(s) = P_2 + v_2(s)$$



Intersection occurs when

$$P_1 + v_1(t) = P_2 + v_2(s)$$

$$\text{or } (v_1 \times v_2)t = (P_2 - P_1) \times v_2$$

subtract  $P_1$  &  $\div$  by  $v_2$

do a dot multiplication by

$$(v_1 \times v_2) \quad \& \quad \div \text{ by } |v_1 \times v_2|^2$$

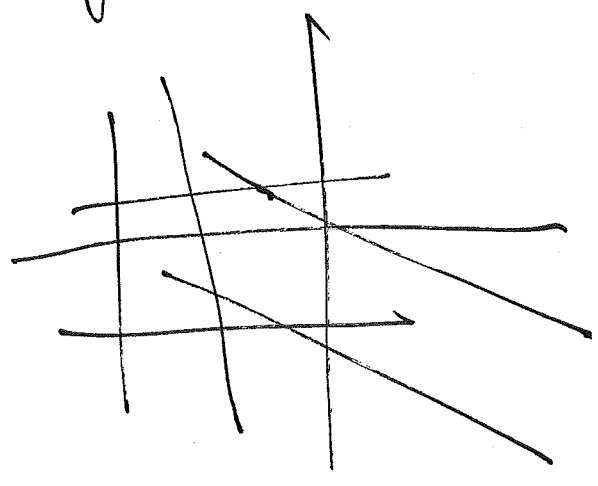
$$t = \frac{\text{Det} \{ (P_2 - P_1), v_2, v_1 \times v_2 \}}{|v_1 \times v_2|^2}$$

if lines are  $\parallel$ , denom  $v_1 \times v_2 = 0$

# Line - Box Intersections

→ Assume it to be an OBB.

↳ consider the 2D case, a line (ray) can intersect a box in 3, same



first common sense  
 → i.e. if any component ( $x$  or  $y$ ) = 0, for ray then no intersection in that direction

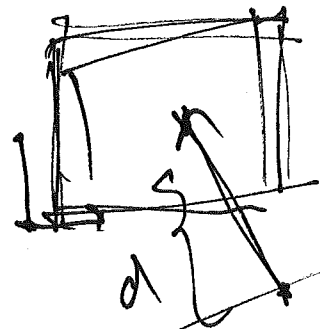
↳ for each axis

$$t_1 = (x_{min} - x_0) / dx$$

$$t_2 = (x_{max} - x_0) / dx$$

repeat for  $y$  &  $z$ .

# Box Plane Intersection



Intersection is returned true

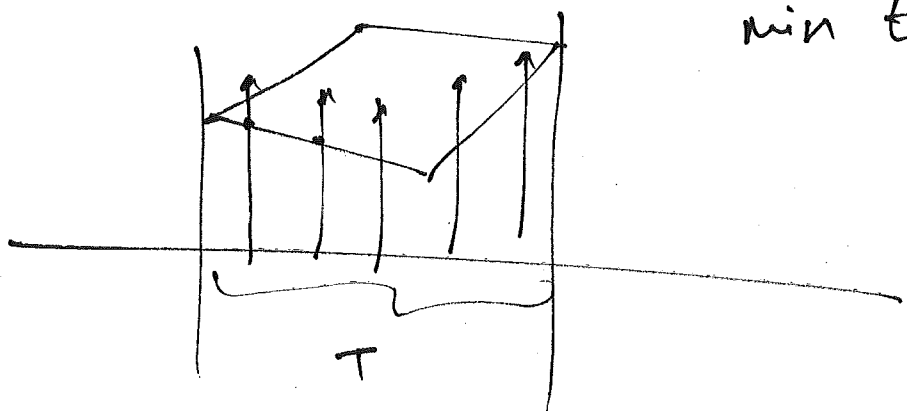
iff

$$|d| \leq a_1 |n \cdot A^1| + a_2 |n \cdot A^2| + a_3 |n \cdot A^3|$$

perp dist

$A^1 A^2 A^3$  are the local axis

$a_1 a_2 a_3$  are dimensions of the box in those directions.



min t value.

