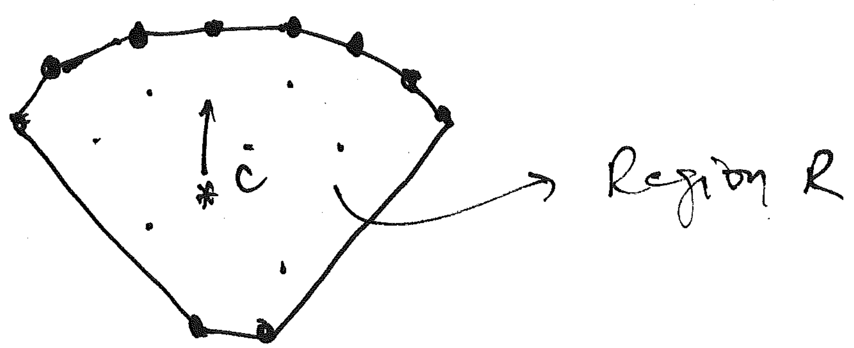
Centroid and Center of Mass

→ If ~~centre~~ density is uniform AND object is convex, then centroid and center of mass will co-incide.

→ Typical calculation of centroid = thro' average of vertices

What will happen in this case?



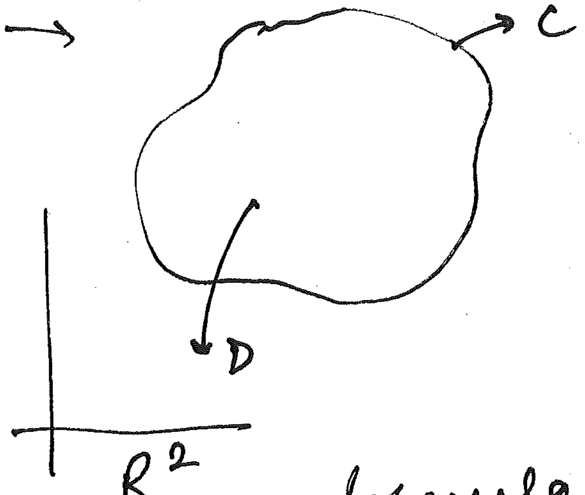
As a general case, Centroid of planar region R =

$$\bar{x} = \frac{\iint_R x\,dx\,dy}{A} = \frac{M_x}{A}$$

$$\bar{y} = \frac{\iint_R y\,dx\,dy}{A} = \frac{M_y}{A}$$

$$A = \frac{1}{2}\sum_{i=0}^{n-1} a_i \quad , \quad a_i = x_i y_{i+1} - x_{i+1} y_i$$

Green's theorem.



If f and g are functions defined within bounds of

$$\oint_C f\,dx + g\,dy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dx\,dy$$

formula for area can be derived from this as

$$A = \frac{1}{2}\oint_C x\,dy - y\,dx$$

This principle can be applied to previous case to derive. [If $f = 0$ & $g = \frac{1}{2}x^2$]

$$M_x = \frac{1}{2}\oint_C x^2\,dy$$

or

$$M_x = \frac{1}{6}\sum_{i=0}^{n-1} (x_{i+1} + x_i)\cdot a_i$$

and similarly

$$M_y = \frac{1}{6}\sum_{i=0}^{n-1} (y_{i+1} + y_i)\cdot a_i$$

SNOOKER PHYSICS

→ Ball - Ball, Ball - Table and Cue - Ball intersection / response

→ Types of collision
- plane - sphere
- sphere - sphere
- sphere - line.

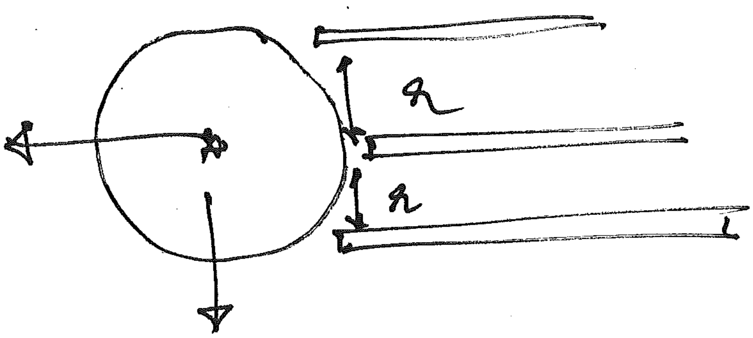
→ Forces

→ Force of cue on ball

$$F_{cb} \approx \Delta x$$ { amount cue is drawn

or $F_{cb} = k \Delta x$ { actually equal to both velocity & distance

→ Effect on ball



$$F = m a_b$$

$$-k \, x_c = m a_b$$

(we are interested in x & $\dot{x}$

$$\Delta \vec{V}_b = \frac{-k \Delta t}{m} x_c$$

→ This gives translation, what about ~~~~ rotation?

we know that

$$\tau = I_b \alpha_b \qquad I_b = 2/5\, MR^2$$

$$\tau = r \times F_c \qquad \text{from prev case}$$

or

$$r \times (-k x_c) = \frac{2}{5} MR^2 \frac{\Delta \dot{\omega}_b}{\Delta t}$$

or

$$\Delta \omega = \frac{5}{2}\left(\frac{\Delta t}{MR^2}\right)\left(r \times (-k x_c)\right)$$

Once in motion :-

- for sliding motion, condition is

$$v_p = R \cdot \vec{\omega}_b$$

or

$$v_p = (\omega \times R) + v$$

→ for friction

$$F_{fb} = \mu m_b g$$

adding direction

$$F_{fb} = -\mu \cdot mg \frac{\vec{V_p}}{|V_p|}$$

for free motion

$$F_b = ma$$

$$-\mu mg \frac{V_p}{|V_p|} = m \frac{V_b}{\Delta t}$$

$$\text{or } V_b = -\mu g \frac{V_p}{|V_p|} \Delta t$$

$|||^{ly}$ we can get

$$\Delta w = \frac{5}{2} \left[\vec{r} \times \left(-\mu mg R \frac{V_p}{|V_p|} \right) \right] \frac{\Delta t}{mR^2}$$

→ Collisions (Ball - Ball | Ball - Table)

Ball - Ball : normal at point of collision

$$\hat{n} = \frac{\vec{r_1} - \vec{r_2}}{|\bar{r_1} - \bar{r_2}|}$$

reduce velocity vector to normal and tangent components

$$\vec{V_{n1}} = [V_1 \cdot \hat{n}]\,\hat{n}$$

$$\vec{V_{t1}} = \vec{V_{n1}} - \vec{V_1}$$

Depending on collision type (for completely elastic, velocities are exchanged along normal direction, tangent vel remains constant)

or $\vec{V_2}' = \vec{V_{t1}} + \vec{V_{n2}}$ etc.

for rotational { when collision occurs a net torque is applied }

$$\vec{V_{pR}} = \vec{V_{p_2}} - \vec{V_{p_1}}$$

vector to point of contact

$$\sim (\underbrace{\vec{r_1 \times W_2}}_{= -r_2 \times W_2}) - (\overrightarrow{r_2 \times W_1}) \quad \Big\{$$

→ Need to calculate friction force @ this point of contact

7

$$f_f = \mu \underbrace{m \frac{\Delta v_{n_1}}{\Delta t}}_{\text{normal force}} \cdot \underbrace{\frac{\vec{V_{PR}} + \vec{V_{t_2}}}{|\vec{V_{PR}} + \vec{V_{t_2}}|}}_{\text{direction}}$$

$$\tau_f = I\alpha$$

$$r \times F_f = I\alpha$$

or $$w_2 = \frac{5}{2}\left[\vec{r_2} \times \left[\cancel{\mu} F_f\right]\right]\frac{\Delta t}{MR^2}$$

Source: google Brian Townsend + physics snooker

for all balls

! Initialize

→ Collision b/n ball & cue

↳ update physics for white ball $\begin{bmatrix} v, & w \\ r, & \theta \end{bmatrix}$

→ Collision b/n all other

→ Run physic loop for free motion