Closed-loop supply chain models for a high-tech product under alternative reverse channel and collection cost structures

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Abstract

In this paper we study closed-loop supply chain models for a high-tech product which is featured with a short life-cycle and volatile demand. We focus on the manufacturer’s choice of three alternative reverse channel structures for collecting the used product from consumers for remanufacturing: (1) the manufacturer collects the used product directly; (2) the retailer collects the used product for the manufacturer; and (3) the manufacturer subcontracts the used product collection to a third-party firm. We characterize and compare the manufacturer’s optimal production quantities and profits under the three alternative reverse channel structures. We also investigate the impacts of collection cost structures and implementations of product take-back laws on the manufacturer’s choice of reverse channel structures.

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1. Introduction

With the fast developments in product remanufacturing to improve economic and environmental performance, an increasing number of manufacturers in the automobile, machinery, appliances, electronics, personal computers, etc., are offering remanufactured goods and associated services. It is estimated that $100 billion of remanufactured goods are sold each year in the U.S. and more than 500,000 people are employed in the remanufacturing industry (Hagerty and Gla, 2011).

In this paper, we focus on the remanufacturing of a high-tech product (e.g., GPS, cell phones, MP3 players, computers, digital camera, and video game systems) which is featured with a short life-cycle and volatile demand due to rapid technology innovation and frequent new product introductions. If the manufacturer is unwilling to collect the used product, consumers often discard the obsolete high-tech product when a technically advanced version is introduced to the market. For example, it is estimated consumers in the United States scrap 400 million electronic products per year (Daly, 2006). To minimize the amount of electronic waste (e-waste) that goes into landfills and to save the cost for materials, many high-tech manufacturers have begun to collect the used product from consumers and explore the value-added product recovery through remanufacturing, whereby worn-out components are replaced, whereas durable components are reused in the making of a remanufactured product.

We next discuss three key features of the closed-loop supply chain (CLSC) for the high-tech product in Section 1.1; provide a literature review and highlight our research contributions in Section 1.2; and summarize our main research findings in Section 1.3.

1.1. Key features of the CLSC for the high-tech product

1.1.1. Reverse channel structures

In a traditional forward-only supply chain, a manufacturer sells the product via a retailer to consumers and does not collect the used product. However, in a CLSC, the manufacturer not only sells the original product to consumers through her forward channel, but also collects the used product for remanufacturing and recycling through her reverse channel. Hence, the choice of an appropriate reverse channel structure is important to the manufacturer’s overall profit in the CLSC.

In practice, there are generally three alternative reverse channel structures that have been deployed by high-tech manufacturers for collecting the used product. First, some high-tech manufacturers collect the used product directly from consumers. For example, Samsung collects televisions, monitors, cell phones, and other consumer electronic products by offering the consumers a free mail-back option and a permanent drop-off option over 200 locations. Second, some high-tech manufacturers collect the used product from consumers through their retailers. For example, Sony has created the GreenFill Program that provides its retailers collection kiosks for used electronics. Similarly, Dell offers consumers free recycling for all its
computers, printers, monitors, and peripheral items for free at Staples. Third, some high-tech manufacturers collect the used product from consumers through a third-party firm. For example, LG Electronics partnered with Waste Management to establish the LG Electronics Recycling Program. Waste Management provides the collection services for LG-brand electronic products.

Motivated by the above reverse channel examples in the high-tech CLSCs, in this paper we consider the manufacturer’s choice of three alternative reverse channel structures: (1) the manufacturer collects the used product directly; (2) the retailer collects the used product for the manufacturer; and (3) the manufacturer subcontracts the used product collection to a third-party firm.

1.1.2. Collection cost structures

There are generally two alternative collection cost structures that have been observed in practice and studied in the CLSC literature. In the first alternative, the collection cost exhibits economies of scale in the total collection volume, i.e., the more used products collected, the lower the per unit collection cost. Such a collection cost structure is more appropriate when the collecting firm uses a simple drop-off collection method, i.e., consumers either drop off the used products to the collection site or mail them back in prepaid mailboxes provided by the collecting firm, whereas the collection firm focuses on efforts to raise consumer awareness of the use product collection program (Atasu et al., 2013; Savaskan et al., 2004). In the second alternative, the collection cost exhibits diseconomies of scale in the total collection volume, i.e., the more used products collected, the higher the per unit collection cost. Such a collection cost structure is more appropriate when the firm uses a relatively more complex pick-up collection method, i.e., the collection firm prefers to collect used products from closer or cheaper sources or more densely populated areas first and it would be more costly to collect additional used products from consumers farther away (Atasu et al., 2013; Ferguson and Tolkay, 2006; Guide, 2003). Motivated by the above collection cost structures, in this paper we study the CLSC models under two alternative collection cost structures with economies and diseconomies of scale.

1.1.3. Take-back laws

Since high-tech products are highly perishable, consumers often distinguish between the original and remanufactured products. The remanufactured high-tech product is often sold at a low margin to less technology-driven or more price-sensitive consumers. As pointed out in Atasu et al. (2013), an e-waste processing cost of up to 3% of the revenue could have a significant impact on the high-tech manufacturer’s profitability. As a result, an economically interested manufacturer often chooses a low used product collection rate which is far from environmentally optimal. This phenomenon is known as the tragedy of the commons dilemma that arises when the common good does not align perfectly with the good of individual entities (Hardin, 1968; Chopra and Meindl, 2013). To overcome this dilemma, product take-back legislation has been popular in recent years, especially for high-tech products that often generate large amount of e-waste to landfills. For example, in Europe, the European Commission has enacted the Waste Electrical and Electronic Equipment (WEEE) Directive (Directive 2003/108/EC) such that European Union member states must establish collection systems for e-waste. In the U.S., there are currently more than 25 states that have passed product take-back legislations mandating statewide e-waste collection and recycling (Electronics Takeback Coalition, 2011). Since the product take-back legislation has played an important role in aligning the high-tech manufacturer’s economic interest with the environmental interest of the public, in this paper we also investigate the impact of the product take-back law on the manufacturer’s choice of reverse channel structures.

1.2. Related literature and research contributions

Recently, we have seen a growing body of research on CLSCs. We refer interested readers to Savaskan and Van Wassenhove (2006), Atasu et al. (2008), and Guide and Van Wassenhove (2009) for complete reviews of this part of literature. Within this research stream, there are two papers that are most closely related to this research. Savaskan et al. (2004) is the first to study the manufacturer’s choice of reverse channel structures for collecting used products from consumers. They assume the collection cost exhibits economies of scale and find that the reverse channel structure with retailer collecting is optimal to the manufacturer. Atasu et al. (2013) further extend Savaskan et al. (2004) by studying the manufacturer’s choice of reverse channel structures under two alternative collection cost structures that exhibit economies and diseconomies of scale. They further show if there are diseconomies of scale in collection cost, then the reverse channel structure with manufacturer collecting is optimal to the manufacturer.

Compared to those two papers, this research offers two main contributions. First, both Savaskan et al. (2004) and Atasu et al. (2013) focus on the CLSC models for a relatively long life-cycle product with deterministic demand. However, in this paper we focus on the CLSC models for a short life-cycle high-tech product with uncertain demand. Our stochastic newsvendor modeling framework is more appropriate for the high-tech product and our model analysis is significantly different from the deterministic modeling framework in those two papers. As far as we know, there is no previous research that uses the newsvendor model to study the manufacturer’s choice of reverse channel structures for the high-tech product in the CLSCs.

Second, both Savaskan et al. (2004) and Atasu et al. (2013) assume that the collection rate of the used product is endogenously determined by the collecting firm. However, in practice, especially for high-tech products that often generate large amount of e-waste to landfills, the collection rate of the used high-tech product is often mandated by the take-back legislation (e.g., WEEE Directive) to avoid the tragedy of the commons dilemma that arises when an economically interested high-tech manufacturer chooses a low used product collection rate that is far from environmentally optimal. In view of this, in this paper we assume that the collection rate is exogenously mandated by the take-back law. As far as we know, there is no previous research that studies the impact of the product take-back law on the manufacturer’s choice of reverse channel structures in the CLSCs.

Finally, our research is also closely related to the newsvendor literature on the traditional forward-only supply chains for a short life-cycle product with uncertain demand. We refer interested readers to Cachon (2003) for a complete review of this part of literature. Within this research stream, this paper is most closely related to Lariviere and Porteus (2001) who study a forward-only supply chain comprised of a single manufacturer and a single retailer selling a short life-cycle product with uncertain demand. This research extends Lariviere and Porteus (2001) by introducing three types of reverse channel structures to the supply chain and show how the addition of a reverse channel will affect the manufacturer’s forward channel decisions and overall profits in the CLSCs.

1.3. Main research findings

In summary, in this paper we study both centralized and decentralized CLSC models for a short life-cycle high-tech product with uncertain demand under three alternative reverse channel structures. We also investigate the impact of collection cost structures and product take-back laws on the manufacturer’s choice of reverse channel structures. This paper provides the following main research
findings for the short life-cycle high-tech product with uncertain demand.

First, we find that if the collection cost structures of the retailer, manufacturer, and third-party firm are symmetric, then the reverse channel structure with retailer collecting is optimal to the manufacturer under economies of scale in collection cost, but the reverse channel structure with manufacturer collecting is optimal to the manufacturer under diseconomies of scale in collection cost. This result is consistent with Savaskan et al. (2004) and Atasu et al. (2013). We also find that this result is generally robust no matter whether the quantity of collected used products is a fraction of the retailer's order quantity from the manufacturer or sales quantity in the regular season.

Second, we find that if the collection cost structures of the retailer, manufacturer, and third-party firm are asymmetric, then the manufacturer's choice of reverse channel structures may deviate from Savaskan et al. (2004) and Atasu et al. (2013). Specifically, we find that if the retailer's collection cost is sufficiently high, then the reverse channel structure with manufacturer collecting is optimal to the manufacturer under economies of scale in collection cost. If the manufacturer's collection cost is sufficiently high, then the reverse channel structure with retailer collecting is optimal to the manufacturer under diseconomies of scale in collection cost.

Finally, this research offers additional insights on the impact of the product take-back law on the manufacturer's forward channel decision and overall profit in the CLSC. Specifically, we find that if there are economies of scale in collection cost, then the legislator could set a high collection rate target in the take-back law since a higher collection rate will not only improve the environmental performance, but also increase the manufacturer's forward channel production quantity and overall profit, and make the high-tech product more available to end consumers. However, if there are diseconomies of scale in collection cost, then the legislator needs to realize that if the collection rate target in the take-back law is set too high (i.e., above a threshold level), it will decrease the manufacturer's forward channel production quantity and overall profit, and make the high-tech product less available to end consumers.

The rest of the paper is organized as follows. In Section 2 we analyze the CLSC models. In Section 3, we compare the manufacturer's production quantities and profits among the decentralized CLSC models. We discuss model extensions in Section 4. Finally, Section 5 concludes with a summary.

2. Closed-loop supply chain models

Following are the notations and assumptions for our model analysis:

- $p_r$: retail price per unit of the remanufactured product;
- $c$: manufacturer's production cost per unit of the product;
- $\Delta$: i.e. margin per unit of the remanufactured product;
- $C_r(r, Q)$: total collection cost when $rQ$ units of the product are collected;
- $A$: a scale parameter that measures the costliness of collecting in $C_r(r, Q)$ ($A > 0$);
- $B$: variable collection cost in $C_r(r, Q)$ ($B > 0$).

**Assumption 1.** $0 \leq v < c < w < p$.

Assumption 1 is used to avoid trivial solutions.

**Assumption 2.** $F(x)$ is differentiable, invertible, and strictly increasing over $[0, \infty)$ and its generalized failure rate $g(x)$, defined as $g(x) = xF(x)/(1 - F(x))$, is increasing.

Assumption 2 is commonly used in the newsvendor supply chain models under a wholesale price-only contract (e.g., Lariviere and Porteus, 2001). The increasing general failure rate (IGFR) is satisfied by some commonly used demand probability distributions such as uniform and normal distributions.

**Assumption 3.** $A < \Delta < A + (c - v)/r$.

We assume that the margin of the remanufactured product $\Delta$ is at least higher than the variable collection cost of the used product $A$, but cannot be too profitable so that it pushes the firm's forward channel to supply unlimited products in the market.

**Assumption 4.** The collection cost function $C_r(r, Q)$ is in one of the following two alternative forms:

$$C_r(r, Q) = \begin{cases} C_1(r, Q) = Ar + B Br^2, \\ C_0(r, Q) = Ar + B Br^2. \end{cases}$$

(1)

Assumption 4 defines two alternative collection cost structures that have been observed in practice and used in the closed-loop supply chain literature. Specifically, if $C_1(r, Q) = C_0(r, Q)$, then the collection cost structure exhibits economies of scale, i.e., the per unit collection cost, $\frac{C_0(r, Q)}{Q} = \frac{Ar + B Br^2}{Q}$, is decreasing in the total quantity of collected used products $rQ$, or equivalently, in the total production quantity of the original product $Q$ such that $C_0(r, Q)$ is a decreasing function of the original product $Q$ such that $C_0(r, Q)$ is a decreasing function of the original product $Q$. Such a collection cost structure has been used in Savaskan et al. (2004). If $C_1(r, Q) = C_0(r, Q)$, then the collection cost structure exhibits diseconomies of scale, i.e., the per unit collection cost, $\frac{C_1(r, Q)}{Q} = \frac{Ar + B Br^2}{Q}$, is increasing in the total quantity of the collected used products $rQ$, or equivalently, in the total production quantity of the original product $Q$. Such a collection cost structure has been used in Atasu et al. (2013).

**Assumption 5.** The collection rate, $r$, is exogenous.

We assume that the collection rate is mandated under the take-back law, which requires that firms take responsibility for the collection of their products. This assumption is reasonable for the high-tech product. In practice, the mandatory collection rate targets in the take-back laws tend to be set sufficiently high in order to force the high-tech manufacturers to take extra efforts collecting more used products for better environmental performance (Atasu et al., 2013). For example, the newly proposed WEEE Directive in 2008 calls for 65% collection rate target (by weight) to be reached by 2016. In the U.S., the collection rate targets for consumer electronics are about 60–80% in states such as Indiana, Michigan, Minnesota, and Wisconsin. This exogenous collection rate assumption has been used in the CLSC literature (see, e.g., Ferrer and Swaminathan (2010) and Webster and Mitra (2007)). To gain more insights, in Section 4.1, we relax this assumption and assume the collection rate is endogenously determined by the collecting firm.
2.1. Model C: centralized closed-loop supply chain

We begin by investigating a centralized CLSC in which an integrated firm (manufacturer) owns her retail channel and acts as a central planner for the supply chain. The sequence of events of Model C is depicted in Fig. 1.

The integrated firm will choose an optimal production quantity of the original product to maximize her expected profit function \( \pi^e(Q) \) given by

\[
\pi^e(Q) = \int_0^Q (p_x + v(Q-x)) f(x) \, dx + \int_Q^\infty p Q f(x) \, dx - c Q
\]

forward channel profit

reverse channel profit

(2)

From (2), the integrated firm’s expected profit is comprised of her forward channel’s expected profit for manufacturing and selling the original product and her reverse channel’s profit for collecting, remanufacturing, and reselling the remanufactured product.

**Theorem 1.** There exists a unique and finite optimal production quantity \( Q^C \) that maximizes the integrated firm’s optimal profit in Model C:

(i) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then \( Q^C \) satisfies the following first-order optimality condition:

\[
\frac{p - c - (p - v) F(Q^C)}{\tau} = 0.
\]

forward channel economic trade-off reverse channel economic trade-off

(3)

(ii) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then \( Q^C \) satisfies the following first-order optimality condition:

\[
\frac{p - c - (p - v) F(Q^C)}{\tau} - \frac{\Delta A}{2} Q^C = 0.
\]

forward channel economic trade-off reverse channel economic trade-off

(4)

**Proof.** All proofs are provided in Appendix A.

Theorem 1 characterizes the integrated firm’s optimal production quantities of the original product in her forward channel under the two alternative collection cost structures in her reverse channel. In general, the optimal production quantity \( Q^C \) reflects the balance between the integrated firm’s economics trade-offs in his forward and reverse channels.

First, if the collection cost exhibits economies of scale, then the forward channel’s economic tradeoff, captured by the first term of (3), is the classical newsvendor critical fractile, which is the integrated firm’s optimal service level (probability) of not stocking out. However, if \( \tau > 0 \), then there is another type of economic trade-off, captured by the second term of (3), which reflects the balance between the reverse channel’s marginal benefit \( \Delta r \) and marginal cost \( Ar \) if one more unit of the original product is manufactured in the forward channel. Since \( \Delta > A \) by Assumption 3, the reverse channel’s profit is always increasing in the forward channel’s production quantity. As a result, the reverse channel will push the forward channel to produce more original products.

Second, if the collection cost exhibits diseconomies of scale, then from the first term of (4), we see the same classical newsvendor trade-off in the integrated firm’s forward channel. However, from the second term of (4), we see that the reverse channel’s marginal benefit \( \Delta r \) is the same, but the marginal cost, \( Ar + B\tau^2 Q \), is increasing in the forward channel’s production quantity \( Q \) due to diseconomies of scale in collection cost.

**Proposition 1.** (i) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then \( Q^C > Q^F \). (ii) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then there exists a threshold collection rate \( \tau^t = (\Delta - A)/(BQ^C) \) so that if \( \tau < \tau^t \), then \( Q^C > Q^F \) whereas if \( \tau > \tau^t \), then \( Q^C < Q^F \).

Proposition 1(i) indicates that if the collection cost exhibits economies of scale, then the integrated firm’s optimal production quantity in Model C is higher than the forward-only supply chain. However, when the collection cost exhibits diseconomies of scale, Proposition 1(ii) shows that if the collection rate is lower than a threshold level, then the integrated firm’s optimal production quantity in Model C is higher than the forward-only supply chain since the reverse channel is profitable and pushes the forward channel to produce more. However, if the collection rate is higher than the threshold level, then the integrated firm’s optimal production quantity in Model C will be lower than the forward-only supply chain since the reverse channel becomes unprofitable and pushes the forward channel to produce less.

**Proposition 2.** (i) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then \( dQ^C/d\tau > 0 \). (ii) if \( C_i(\tau, Q) = C_0(\tau, Q) \), then there exists a threshold quantity \( Q^C \) so that if \( Q^C < Q^C \), then \( dQ^C/d\tau \geq 0 \) whereas if \( Q^C > Q^C \), then \( dQ^C/d\tau < 0 \).

Proposition 2 further shows the impact of the collection rate \( \tau \) on the optimal production quantities \( Q^C \) under the two collection cost structures. When there are economies of scale in the collection cost structure, a higher collection rate \( \tau \) will always increase the optimal production quantity in Model C. However, when there are diseconomies of scale in the collection cost structure, \( Q^C \) will be increasing in the collection rate \( \tau \) as long as it is below a threshold production quantity \( Q^C \), but will be decreasing in \( \tau \) when it is above the threshold value. This result could be explained by the fact that the economic trade-off in the forward channel is independent of the collection rate \( \tau \), but the economic trade-off in the reverse channel is dependent upon it. If the optimal production quantity is below (above) the threshold value, then as \( \tau \) increases, the reverse channel’s marginal benefit \( \Delta r \) is increasing (decreasing).
2.2. Model M: closed-loop supply chain with manufacturer collecting

In this subsection, we investigate a decentralized CLSC comprised of an independent manufacturer who acts as the Stackelberg leader and an independent retailer who acts as the Stackelberg follower. The retailer is responsible for selling the original product to consumers whereas the manufacturer is responsible for collecting the used product from consumers. The sequence of events of Model M is depicted in Fig. 2.

In the manufacturer–leader–retailer–follower game of Model M, we work backward starting with the retailer’s expected profit function $\pi^r(Q)$ given by

$$\pi^r(Q) = \int_0^Q [px + v(Q - x)]f(x) \, dx + \int_Q^\infty pQf(x) \, dx - wQ.$$

For a given wholesale price $w$, it follows from the classical news-vendor model that the retailer’s optimal order quantity $Q^*(w)$ is uniquely given by

$$Q^*(w) = F^{-1}[(p - w)/(p - v)].$$

The manufacturer anticipates the retailer’s optimal order quantity $Q^*(w)$ and will choose an optimal wholesale price $w^M$ to maximize his profit function $\pi^M(w)$ given by

$$\pi^M(w) = (w - c)Q^*(w) + \Delta Q^*(w) - C_l(r, Q^*(w)).$$

To identify the manufacturer’s optimal wholesale price $w^M$, we use an alternative expression for the manufacturer’s profit. From (6), there is a one-to-one relationship between $Q^*(w)$ and $w$. Similar to Lariviere and Porteus (2001), we work with an equivalent formulation in which the manufacturer faces the following inverse function of $Q^*(w)$:

$$w^*(Q) = p - (p - v)F(Q).$$

So instead of working with $w$, we work with $Q$ to identify the optimal order quantity that maximizes the manufacturer’s profit. Then we can rewrite the manufacturer’s profit function (7) as follows:

$$\pi^M(Q) = \frac{[p - (p - v)v(Q) - c]Q}{\text{manufacturer’s forward channel profit}} + \frac{\Delta Q - C_l(r, Q)}{\text{manufacturer’s reverse channel profit}}.$$  

**Theorem 2.** There exists a unique and finite optimal order quantity $Q^M$ that maximizes the manufacturer’s expected profit in Model M:

(i) If $C_l(r, Q) = C_p(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

$$p - v[F(Q^M) - g(Q^M)] = 0,$$

manufacturer’s forward channel economic trade-off

(ii) If $C_l(r, Q) = C_p(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

$$p - v[F(Q^M) - g(Q^M)] - (c - v) = 0,$$

manufacturer’s reverse channel economic trade-off

The manufacturer collects the used product from consumers. The manufacturer makes the remanufactured product for resale. The retailer sells the unsold products after the season. The retailer salvages the unsold products after the season. The retailer orders the product from the manufacturer before the season. Selling season Clearance season Collection period Remanufacturing period

**Fig. 2.** Sequence of events of Model M.
fewer products in her forward channel than the traditional forward-only supply chain due to the unfavorable business condition in her reverse channel.

**Proposition 4.** (i) If \( C_t(Q, w) = C_e(Q, w) \), then \( dQ^M/d\tau > 0 \).
(ii) If \( C_t(Q, w) = C_e(Q, w) \), then there exists a threshold quantity \( Q^M = Q^m \) so that if \( Q^M \leq Q^m \), then \( dQ^M/d\tau \geq 0 \) whereas if \( Q^M > Q^m \), then \( dQ^M/d\tau < 0 \).

Proposition 4 further shows the impact of the collection rate \( \tau \) on the optimal order quantity \( Q^* \). In brief, when there are economies of scale in collection cost, a higher collection rate \( \tau \) will always increase the optimal order quantity. However, when there are diseconomies of scale in collection cost, the optimal order quantity is increasing in the collection rate \( \tau \) if it is below a threshold order quantity, but is decreasing in \( \tau \) when it is above the threshold value.

2.3. Model R: closed-loop supply chain with retailer collecting

In this subsection, we investigate a decentralized CLSC comprised of an independent manufacturer who acts as the Stackelberg leader and an independent retailer who acts as the Stackelberg follower in the CLSC. The retailer is not only responsible for selling the original product to consumers but also responsible for collecting the used product from consumers for the manufacturer. The sequence of events of Model R is depicted in Fig. 3.

In the manufacturer–leader–retailer–follower game of Model R, we work backward starting with the retailer’s expected profit function \( \tilde{x}^R(Q) \) given by

\[
\tilde{x}^R(Q) = \int_0^Q [p + v(Q - x)]f(x) \, dx + \int_Q^\infty pQf(x) \, dx - wQ \\
+ \frac{brQ - C_t(Q, w)}{C_t(Q, w)} \\text{--- retailer’s expected profit in forward channel} \\
+ \frac{brQ - C_t(Q, w)}{C_t(Q, w)} \\text{--- retailer’s profit in reverse channel}
\]

**Lemma 1.** For given wholesale price and buyback price \((w, b)\), there exists a unique and finite optimal order quantity \( Q^*(w, b) \) that maximizes the retailer’s expected profit in Model R:

(i) if \( C_t(Q, w) = C_e(Q, w) \), then \( Q^*(w, b) \) satisfies the following first-order condition:

\[
p - w - (p - v)Q^*(w, b) + (b - A)\tau = 0.
\]

(iii) if \( C_t(Q, w) = C_e(Q, w) \), then \( Q^*(w, b) \) satisfies the following first-order condition:

\[
p - w - (p - v)Q^*(w, b) + (b - A)\tau - Br^2Q^*(w, b) = 0.
\]

**Lemma 1** shows that the retailer’s optimal response \( Q^*(w, b) \) to the manufacturer’s contract \((w, b)\) in Model R also reflects the balance between the retailer’s economic trade-offs in his forward and reverse channels. This contrasts with the retailer in Model M, whose optimal response to the manufacturer’s wholesale price \( w \) only reflects the retailer’s classical newsvendor tradeoff in his forward channel.

The manufacturer anticipates the retailer’s optimal order quantity \( Q^*(w, b) \) and will choose an optimal wholesale price and an optimal buyback price to maximize her profit function \( \tilde{x}^M(w, b) \) given by

\[
\tilde{x}^M(w, b) = \left( w - Q^*(w, b) \right) \text{manufacturer’s forward channel profit} + \left( A - b \right) Q^*(w, b) \text{manufacturer’s reverse channel profit}.
\]

To identify the manufacturer’s optimal wholesale price \( w^* \), we use an alternative expression for the manufacturer’s profit. From (15) and (16), there is a one-to-one relationship \( Q^*(w, b) \) and \( w \). We work with an equivalent formulation in which the manufacturer faces the following inverse function of \( Q^*(w, b) \):

\[
w^*(Q, b) = \left\{ \begin{array}{ll}
p - (p - v)Q + (b - A)\tau & \text{if } C_t(Q, w) = C_e(Q, w) \\
p - (p - v)Q + (b - A)\tau - Br^2Q & \text{if } C_t(Q, w) = C_e(Q, w).
\end{array} \right.
\]

So instead of working with \( w \), we work with \( Q \) to identify the optimal order quantity that maximizes the manufacturer’s profit. Then we can rewrite the manufacturer’s profit function (17) as follows:

\[
\tilde{x}^M(Q) = \left\{ \begin{array}{ll}
p - c - (p - v)Q + (\Delta - A)\tau Q & \text{if } C_t(Q, w) = C_e(Q, w) \\
p - c - (p - v)Q + (\Delta - A)\tau - Br^2Q & \text{if } C_t(Q, w) = C_e(Q, w).
\end{array} \right.
\]

Interestingly, from (19) we see that the manufacturer’s profit function is independent of the buyback price \( b \). Hence, the manufacturer will choose an optimal order quantity to maximize her profit.

**Theorem 3.** There exists a unique and finite optimal order quantity \( Q^e \) that maximizes the manufacturer’s expected profit in Model R:

(i) if \( C_t(Q, w) = C_e(Q, w) \), then \( Q^e \) satisfies the following first-order optimality condition:

\[
p - v[1 - F(Q^e)] + (\Delta - A)\tau - (c - v) = 0.
\]

(ii) if \( C_t(Q, w) = C_e(Q, w) \), then \( Q^e \) satisfies the following first-order optimality condition:

\[
p - v[1 - F(Q^e)] + (\Delta - A)\tau - Br^2Q^e = 0.
\]
Theorem 3 characterizes the retailer’s optimal order quantity $Q^w$, or equivalently, the manufacturer's optimal wholesale price and optimal buyback price $(w^R, b^R)$ that are given by

$$w^R = \begin{cases} p - (p - v)F(Q^R) + (b^R - A)\tau & \text{if } C(t, Q) = C(t, Q), \\ p - (p - v)F(Q^R) + (b^R - A)\tau - Br^2Q & \text{if } C(t, Q) = C(t, Q) \end{cases}$$

(22)

From (20) and (21), we see that the optimal order quantity $Q^R$ reflects the balance between the manufacturer's economic trade-offs in her forward and reverse channels. The first type of economic trade-off, captured by the first terms in (20) and (21), is still the manufacturer's economic trade-off under a wholesale price-only contract when selling to a newsvendor retailer. If $\tau = 0$ (i.e., there is no reverse channel), then the optimal order quantity and the optimal wholesale price reduce to the manufacturer's optimal order quantity $Q^R = Q^M$ and $w^R = w^M$ under a wholesale price-only contract in a forward-only channel, where $Q^M$ is given by (12) and $w^M$ is given by (13).

The second type of economic trade-off in Model R, captured by the second terms in (20) and (21), reflects the balance between the reverse channel's marginal benefit and marginal cost if one more unit of the product is made in the forward channel. In brief, if there are economies of scale in collection cost, then the reverse channel's economic trade-off in Model R is exactly the same as that in Model M. However, if there are diseconomies of scale in collection cost, then from (21) we see that while the reverse channel's marginal benefit $\Delta \tau$ in Model R is still the same as in Model M, the reverse channel's marginal cost $A + 2Br^2Q$ in Model R is higher than the reverse channel's marginal cost $A + Br^2Q$ in Model M.

**Proposition 5.** (i) If $C(t, Q) = C(t, Q)$, then $Q^R > Q^M$. (ii) If $C(t, Q) = C(t, Q)$, then there exists a threshold collection rate $\tau^* = \frac{Q^M}{Q^R}$ so that if $\tau < \tau^*$, then $Q^R \geq Q^M$ whereas if $\tau > \tau^*$, then $Q^R < Q^M$.

Proposition 5(i) shows that when the collection cost exhibits economies of scale, the manufacturer in Model R will choose an optimal order quantity that is higher than the traditional forward-only channel. However, when the collection cost exhibits diseconomies of scale, Proposition 5(ii) shows that if the collection rate is lower than a threshold level, then the manufacturer in Model R will produce more products in her forward channel than the traditional forward-only supply chain to take advantage of her reverse channel's favorable business condition whereas if the collection rate is higher than the threshold level, then the manufacturer in Model R will produce fewer products in her forward channel than the traditional forward-only supply chain due to the unfavorable business condition in her reverse channel.

**Proposition 6.** (i) If $C(t, Q) = C(t, Q)$, then $dQ^R/d\tau > 0$. (ii) If $C(t, Q) = C(t, Q)$, then there exists a threshold quantity $Q^* = (\Delta - A)/(4B\tau)$ so that if $Q^R \leq Q^*$, then $dQ^R/d\tau \geq 0$ whereas if $Q^R > Q^*$, then $dQ^R/d\tau < 0$.

Proposition 6 further shows the impact of the collection rate $\tau$ on the optimal order quantities in Model R under the two collection cost structures. When there are economies of scale in collection cost, a higher collection rate will always increase the optimal order quantity. However, when there are diseconomies of scale in collection cost, the optimal order quantity is increasing in $\tau$ if it is below a threshold quantity, but is decreasing in $\tau$ if it is above the threshold value. Interestingly, we see that the threshold quantity in Model R happens to be a half of the threshold quantity in Model C and Model M, i.e., $Q^R = \frac{Q^M}{2}$.

2.4. Model 3P: closed-loop supply chain with third-party firm collecting

Finally, we investigate a decentralized CLSC comprised of an independent manufacturer, an independent retailer, and a third-party firm. The manufacturer acts as the leader and the retailer and the third-party firm act as followers in the supply chain. The retailer is only responsible for selling the high-tech product to consumers whereas the third-party firm is fully responsible for collecting the used product for the manufacturer, Fig. 4.

The manufacturer–leader–retailer–follower game in Model 3P is exactly the same as Model M. Hence, for a given wholesale price $w$, the retailer's optimal order quantity $Q^*(w)$ is given by (6).

We next consider the manufacturer–leader–collector–follower game in the reverse channel, by working backward starting with the collector's profit function. Given the retailer's optimal order quantity $Q^*(w)$ and the manufacturer's buyback price $b$, the third-party collector's profit function $\pi_{3P}(w, b)$ is given by

$$\pi_{3P}(w, b) = b^RQ^*(w) - C(t, Q^*(w)).$$

(23)

Let $\pi_{3P} \geq 0$ be the third-party collector's reservation profit, meaning that the collector will only collect the used product for the manufacturer if $\pi_{3P}(w, b) \geq \pi_{3P}$.

The manufacturer anticipates the retailer's optimal order quantity $Q^*(w)$ and the third-party firm's profit function $\pi_{3P}(w, b)$ and reservation profit $\pi_{3P}$, will choose an optimal wholesale price $w^{3P}$ to the retailer and an optimal buyback price $b^{3P}$ to the third-party collector to maximize her profit function $\pi_{M}(w, b)$ given by

$$\pi_{M}(w, b) = \begin{cases} (w - c)Q^*(w) & \text{if } \text{manufacturer's profit in forward channel} \\ (\Delta - b)\tau Q^*(w) & \text{if } \text{manufacturer's profit in reverse channel} \end{cases}.$$  

(24)

For a given $w$, since the manufacturer's profit $\pi_{M}(w, b)$ is strictly decreasing in $b$ and the third-party collector's profit $\pi_{3P}(w, b)$ is strictly increasing in $b$, the manufacturer will choose an optimal buyback price $b^*(w)$ that satisfies $\pi_{M}(w, b^*(w)) = \pi_{3P}$ and is given by

$$b^*(w) = \frac{\pi_{3P} + C(t, Q^*(w))}{\tau Q^*(w)}.$$  

(25)

To identify the manufacturer's optimal wholesale price $w^{3P}$, we work with an equivalent formulation in which the manufacturer...
faces the following inverse functions of $Q^*(w)$ and $b^*(w)$:

$$w(Q) = p - (p - v)\bar{F}(Q).$$

and

$$b(Q) = \frac{\pi_{3P} + C(\tau, Q)}{\tau Q}.$$ 

Then we can rewrite the manufacturer’s profit function expressed in (24) as follows:

$$\tilde{\pi}^M(Q) = \{p - c - (p - v)\bar{F}(Q)\}Q + \Delta\tau Q - C(\tau, Q) - \pi_{3P} = \tilde{\pi}^M(Q) - \pi_{3P}.$$

(26)

**Theorem 4.** There exists a unique optimal order quantity $Q^{3P} = Q^M$ that maximizes the manufacturer’s profit in Model 3P.

Theorem 4 shows that the optimal order quantity in Model 3P is exactly the same as that in Model M. However, there are two key differences between Model M and Model 3P. First, in Model 3P, the manufacturer needs to offer the third-party collector a buyback price $b^{3P} = (\pi_{3P} + C(\tau, Q^{3P})/\tau Q^{3P})$ per unit of the collected used product. Such a transfer payment is not needed when the manufacturer collects the used product by herself in Model M. Second, the manufacturer’s profit in Model 3P is always lower than that in Model M if the third-party collector’s reservation profit level is positive, i.e., $\pi_{3P} > 0$.

3. Comparison of decentralized CLSC models

In this section, we compare the manufacturer’s optimal production quantities and profits among the three decentralized CLSC models in Section 2.

**Theorem 5.** (i) If $C(\tau, Q) = C_\tau(\tau, Q)$, then $Q^M = \bar{Q} = Q^{3P}$. (ii) If $C(\tau, Q) = C_\lambda(\tau, Q)$, then $Q^3 < Q^M = Q^{3P}$.

Theorem 5(i) shows that if there are economies of scale in collection cost, the manufacturer’s optimal production quantities in the three decentralized CLSC models (M, R, and 3P) are exactly the same, due to the fact that the manufacturer’s economic trade-offs in her forward and reverse channels are exactly the same in the three decentralized CLSC models. However, Theorem 5(ii) shows that if there are diseconomies of scale in collection cost, then the manufacturer’s optimal production quantity in Model R will be less than that in Model M and Model 3P, due to the fact that the manufacturer’s marginal cost in her reverse channel is higher in Model R than that in Model M and Model 3P.

**Theorem 6.** (i) If $C(\tau, Q) = C_\tau(\tau, Q)$, then $\tilde{\pi}^{M} (Q^{3P}) \leq \tilde{\pi}^{M} (Q^{M}) < \tilde{\pi}^{M} (Q^{3})$. (ii) If $C(\tau, Q) = C_\lambda(\tau, Q)$, then $\tilde{\pi}^{M} (Q^{3P}) > \tilde{\pi}^{M} (Q^{3})$ and $\tilde{\pi}^{M} (Q^{3P}) \geq \tilde{\pi}^{M} (Q^{3})$.

Theorem 6(i) shows that if there are economies of scale in collection cost, then the manufacturer’s profit in Model R will be the highest, whereas the manufacturer’s profit in Model 3P will be the lowest. However, Theorem 6(ii) shows that if there are diseconomies of scale in collection cost, then the manufacturer’s profit in Model M will be the highest, whereas the manufacturer’s profit in Model R/3P will be the lowest depending upon the third-party firm’s reservation profit level. In summary, Theorem 6 indicates that if there are economies of scale in collection cost, then the manufacturer should choose the reverse channel structure with retailer collecting whereas if there are diseconomies of scale in collection cost, then the manufacturer should choose the reverse channel structure with manufacturer collecting. This research finding is consistent with the results in Savaskan et al. (2004) and Atasu et al. (2013) for a long life-cycle product with deterministic demand.

4. Model extensions

We next discuss some model extensions. We first consider the case that the collection rate is endogenously determined by the collecting firm in Section 4.1. We then study the case that the quantity of the collected used products is a fraction of the retailer’s sales quantity in the regular season in Section 4.2. In Section 4.3, we next study the CLSC models under asymmetric collection costs. Finally, we briefly discuss the impacts of various implementations of tack-back laws in practice on the manufacturer’s reverse channel choice in Section 4.4.

4.1. Endogenous collection rate

To gain more insights, we next conduct a numerical study to compare the manufacturer’s optimal production quantities and profits in the decentralized CLSC models when the collection rate $\tau$ is endogenously determined by the collecting firm. This endogenous collection rate assumption is more reasonable when there is no product take-back law in place (e.g., in some States of the U.S.), or when the remanufactured product is so profitable that the collecting firm will collect more used products than the collection rate target mandated by the take-back law. We use the following combinations of parameters for our numerical study:

\[ p = \{100, 500, 1000\} \quad c = r_1p \quad \text{where} \quad r_1 = \{0.25, 0.75\} \quad v = \tau c \quad \text{where} \quad \tau = \{0.25, 0.75\} \quad B = \{1, 10\} \quad X = \{\text{Uniform, Normal}\} \]

Specifically, we classify the high-tech products into three representative categories based upon their retail prices in the market: low-price ($p=100$), medium-price ($p=500$), and high-price ($p=1000$). These price points are derived by evaluating a range of popular models of consumer electronics and computers available at online sellers such as Apple.com, Dell.com, and Amazon.com. For example, the prices of some popular MP3 players by Sony, SanDisk, and Apple and cell phones by Nokia, Samsung, and Motorola are around $100$. The prices of some popular laptops by Lenovo, Asus, and Toshiba and video game consoles such as PlayStation 4 and Xbox One are around $500$. Finally, the prices of some popular high-end desktop computers such as Dell XPS 18 and Lenovo IdeaCenter are about $1000$. By changing the unit currency, e.g., from 1 dollar to 10 cents or 10 dollars, the retail prices in our numerical study generally can capture a large variety of high-tech products.

In each scenario, the mean demand $\mu$ is fixed at $\mu=100$. For the normal distribution, we select a coefficient of variation $CV = \sigma/\mu = \{0.1, 0.2, 0.3\}$. For the uniform distribution, we choose the range $[0, 200]$ with $\mu = 100$ and $CV \approx 0.577$. Since the manufacturer’s optimal profit in Model M is never lower than that in Model 3P, we focus on the comparison of the manufacturer’s optimal order quantities and profits between Model M and Model R. We vary the collection rate $\tau$ from 0 to 1 with an increment of 0.05 to identify the manufacturer’s optimal collection rates, optimal production quantity, and optimal profit $\{\pi^M, Q^M, \tilde{\pi}^M(Q^M)\}$ in Model M, and the retailer’s optimal collection rate and order quantity, and the manufacturer’s optimal profit $\{\pi^3, Q^3, \tilde{\pi}^3(Q^3)\}$ in Model R.

Tables 1 and 2 below summarize our numerical results. Since we obtain qualitatively similar results at different values of $p = \{100, 500, 1000\}$, $B = \{1, 10\}$, and $CV = \{0.1, 0.2, 0.3\}$, to be concise, we only report our results based upon $p = 100$ and $B = 10$ under the uniform distribution and $p = 100$, $B = 10$, and $CV = \{0.1, 0.3\}$ under the normal distribution.

From Tables 1 and 2, we find that the numerical results are similar between uniform and normal demand distributions.
First, if there are economies of scale in collection cost, then the optimal collection rates in Model M and Model R are exactly the same (i.e., $\gamma^M = \gamma^R = 100\%$), and the reverse channel structure with retailer collecting is optimal to the manufacturer (i.e., $\pi^M(Q^M) < \pi^R(Q^R)$). Interestingly, we find that the collecting firm will chose to collect as many used products as possible (i.e., 100% collection rate) even without the take-back law.

Second, if there are diseconomies of scale in collection cost, then the manufacturer's choice of reverse channel structures generally depends upon the profitability of the original product. Specifically, if the margin of the original product is high (i.e., $\gamma_1$ is low), or the margin of the original product is low but its salvage value is high (i.e., $\gamma_1$ and $\gamma_2$ are high), then the optimal collection rates in Model M and Model R are zero (i.e., $\gamma^M = \gamma^R = 0\%$). The intuition for this result is due to the fact that when there are diseconomies of scale in collection cost, the reverse-channel's collection cost, $C_{ch}(r, Q) = AR_2 + Br(Q)^2 / 2$, is increasing in the forward channel's production quantity $Q$ quickly in a quadratic fashion. If the product is more profitable, the manufacturer will produce more products in the forward channel, which in turn increases the collection cost in the reverse channel to a significant degree that it undermines the forward channel's profit. As a result, the manufacturer or retailer will not collect the used products and the CLSCs in Model M and Model R are exactly the same as the traditional forward-only supply chain.

However, if the margin and salvage value of the original product are low (i.e., $\gamma_1$ is high and $\gamma_2$ is low), then the manufacturer's forward channel production quantity is relatively low and it does not increase the reverse channel's collection cost significantly. As a result, the manufacturer will choose a relatively low collection rate (e.g., $\gamma^M \leq 10\%$ in Model M and $\gamma^M \leq 5\%$ in Model R), and the reverse channel structure with manufacturer collecting is optimal to the manufacturer (i.e., $\pi^M(Q^M) \geq \pi^R(Q^R)$). In summary, our numerical results show that when there are diseconomies of scale in collection cost, an economically interested firm will choose a zero or very low collection rate due to the tragedy of the commons dilemma. Hence, it is important for legislators to implement product take-back laws to force the firms to collect more used products to improve environmental performance.

### 4.2. Quantity of collected used Products is a function of retailer's seasonal sales quantity

In Sections 2 and 3, we assume that the quantity of collected used products, $q$, is a fraction of the retailer's order quantity, $Q$, i.e., $q = qQ$. This assumption is reasonable when the retailer sells all products in the regular season or salvages all unsold products through clearance sales right after the regular season. In practice, however, the retailer may salvage the unsold products in a secondary market (e.g., overseas markets, retail outlets, online auctions such as eBay). It might be infeasible to collect those products sold in a secondary market. In view of this, in this subsection we assume that the quantity of the collected used products, $q$, is a fraction of the retailer's sales quantity, $s$, in the regular season, where $s = \min[X, Q]$. Correspondingly, we redefine the collection cost function $C_{ch}(r, s)$ as follows:

**Assumption 5.** The collection cost function $C_{ch}(r, s)$ is in one of the following two forms:

$$C_{ch}(r, s) = \begin{cases} CE_{ch}(r, s) = ARs + \frac{1}{2}Br^2, \\ Cd_{ch}(r, s) = ARs + \frac{1}{2}Br^2, \end{cases}$$

(27)

From (27), we see demand uncertainty in the forward channel translates into collection cost uncertainty in the reverse channel. The expected collection cost $E[C_{ch}(r, s)]$ can be expressed as

$$E[C_{ch}(r, s)] = \begin{cases} E[CE_{ch}(r, s)] = \int_{0}^{B} 2Arf(x)dx + Az^2(1 - F(Q)) + B\frac{1}{2}r^2, \\ E[Cd_{ch}(r, s)] = \int_{0}^{B} 2Arf(x)dx + Az^2(1 - F(Q)) + B\frac{1}{2}r^2, \end{cases}$$

(28)
We next analyze the three decentralized CLSC models (M, R, 3P) based upon the collection function defined by (27) in Assumption 5. To be concise, we only provide our key results. We refer interested readers to Appendix B for the detailed model analysis and proofs.

**Theorem 7.** (i) If \( C_i(r, s) = C_r(r, s) \), then \( Q^M = Q^A = Q^{3P} \). (ii) If \( C_i(r, s) = C_I(r, s) \), then \( Q^I \leq Q^M = Q^{3P} \).

Theorem 7 is exactly the same as Theorem 5 and suggests that no matter whether the quantity of collected used products is a fraction of the retailer’s order quantity \( Q \) or sales quantity \( s \), if there are economies of scale in collection cost, then the manufacturer’s optimal production quantities in Models M, R and 3P are exactly the same, whereas if there are diseconomies of scale in collection cost, then the manufacturer’s optimal production quantity in Model R is less than that in Model M and Model 3P.

**Theorem 8.** (i) If \( C_i(r, s) = C_I(r, s) \), then \( \Delta M(Q^M) < \Delta M(Q^A) \). (ii) If \( C_i(r, s) = C_I(r, s) \), then \( \Delta M(Q^M) > \Delta M(Q^R) \) and \( \Delta M(Q^M) \geq \Delta M(Q^{3P}) \).

Theorem 8 is exactly the same as Theorem 6 and suggests that no matter whether the quantity of collected used products is a fraction of the retailer’s order quantity \( Q \) or sales quantity \( s \), the reverse channel with retailer collecting is optimal to the manufacturer when there are economies of scale in collection cost whereas the reverse channel with manufacturer collecting is optimal to the manufacturer when there are diseconomies of scales in collection cost.

### 4.3. Asymmetric collection cost Structures

In Sections 2 and 3, we assumed that the collection cost structures of the manufacturer, retailer, and third-party firm are symmetric. However, in practice, the collection costs of those firms may be asymmetric due to their different collection technologies, expertise in collections, and travel distances to consumers, etc. In view of this, in this subsection, we consider the following asymmetric and firm-specific collection cost functions.

#### Assumption 6.

The collection cost function \( C_i(r, Q) \) in Model \( i \) \( \in \{M, R, 3P\} \) is in one of the following two alternative forms:

\[
C_i(r, Q) = \begin{cases} 
C_I(r, Q) = A_Ir + B_Ir^2, & i = M, 3P, \\
C_I(r, Q) = A_rQ + B_rQ^2, & i = R.
\end{cases} 
\]  

Where \( A_I \) is the variable collection cost and \( B_r \) is a scale parameter that measures the costliness of collecting in Model \( i \).

We next analyze the decentralized CLSC models based upon the collection function defined by (28) in Assumption 6. To be concise, we only provide our key results. We refer interested readers to Appendix C for the detailed model analysis and proofs.

**Theorem 9.** For any \( i, j \in \{M, R, 3P\} \) and \( i \neq j \), (i) if \( C_I(r, Q) = C_j(r, Q) \), then \( Q^I > Q^J \) if \( A_I < A_J \). (ii) If \( C_I(r, Q) = C_j(r, Q) \), then \( Q^I > Q^J \) if \( MC_i(r, Q) < MC_j(r, Q) \), where \( MC_i(r, Q) \) is the manufacturer’s marginal collection cost under diseconomies of scale in collection cost in Model \( i \) given by

\[
MC_i(r, Q) = \begin{cases} 
A_I + B_Ir^2, & i \in \{M, 3P\}, \\
A_r + 2BrQ^2, & i = R.
\end{cases} 
\]

Theorem 9 identifies some necessary and sufficient conditions under which the optimal order quantity \( Q^I \) in Model \( i \) will be higher than \( Q^J \) in Model \( j \). Specifically, Theorem 9(i) shows that if there are economies of scale in the collection cost, then the reverse channel structure with a lower variable collection cost will produce more products in the forward channel, due to the fact that the marginal collection cost under economies of scale, \( MC_i(r, Q) = A_Ir \), is increasing in \( A_I \). Theorem 9(ii) shows that if there are diseconomies of scale in collection cost, then the reverse channel structure with a lower marginal collection cost at the other reverse channel’s optimal order quantity will produce more products in the forward channel. Since the marginal collection cost under diseconomies of scale, \( MC_i(r, Q) \), is dependent upon \( A_I, B_I, \) and \( Q \), the comparative results on the manufacturer’s optimal production quantities among the three decentralized CLSCs are jointly determined by the variable collection cost, the costliness of used product collecting, and production quantity \( Q^I \) in the forward channel.

**Theorem 10.** (i) If \( C_I(r, Q) = C_J(r, Q) \), then \( \Delta M(Q^M) > \Delta M(Q^B) \) if \( \Delta M(Q^M) > \Delta M(Q^B) \). (ii) If \( C_I(r, Q) = C_J(r, Q) \), then \( \Delta M(Q^M) > \Delta M(Q^B) \) if \( \Delta M(Q^M) > \Delta M(Q^B) \) and \( \Delta M(Q^M) \geq \Delta M(Q^{3P}) \).

Theorem 10 identifies some sufficient conditions under which the manufacturer’s choices of reverse channel structures under asymmetric collection costs are different from Theorem 6 under symmetric collection costs. Specifically, Theorem 10(i) shows that when there are economies of scale in collection cost, the manufacturer will choose the reverse channel structure with manufacturer collecting instead of retailer collecting in Theorem 6(i) if the retailer’s collection cost is sufficiently higher than the manufacturer’s collection cost at the optimal order quantity \( Q^B \) in Model R. Theorem 10(ii) shows that when there are diseconomies of scale in collection cost, the manufacturer will choose the reverse channel structure with retailer collecting instead of manufacturer collecting in Theorem 6(ii) if the manufacturer’s collection cost is sufficiently higher than the retailer’s collection cost at the optimal order quantity \( Q^M \) in Model M. Finally, Theorem 10(iii) shows that under both economies and diseconomies of scale in collection cost structures, the manufacturer will choose the reverse channel structure with the third-party firm collecting instead of manufacturer collecting if the manufacturer’s collection cost is sufficiently higher than the third-party firm at the optimal order quantity \( Q^M \) in Model M.

### 4.4. Implementations of product take-back laws

So far in this paper we only examine how the collection rate mandated by the product take-back legislation influences the manufacturer’s reverse channel choice. In practice, however, countries often take different approaches implementing product take-back laws. We next discuss some key issues regarding the implementations of take-back laws that may influence the manufacturer’s reverse channel choice.

#### 4.4.1. Individual versus collective producer responsibility

In practice, there are generally two different collection systems under the take-back laws: individual producer responsibility (IPR) and collective producer responsibility (CPR). In this paper, we focus on the IPR system in which manufacturers can create their own collection systems and take responsibility for end-of-life management of their own products. IPR systems have been implemented in many states of the U.S. and some countries in Europe (e.g., Austria, Germany, Italy, and Ireland) and Asia (e.g., Japan). On the other hand, many European Union member states (e.g., Belgium, Denmark, France, Spain, and the U.K.) have implemented the CPR system in which manufacturers take joint responsibility together for the end-of-life management of their products, often through a third-party collector (e.g., municipality collection or contractor collection) and they must pay the collection cost based upon their...
market shares, return shares, or simply the average collection cost. If such a CPR system is implemented, then Model 3P generally applies with some modifications on the collection cost. To illustrate, let $C_i^{3P}$ be the collector's total collection cost in the CPR system. Then for an individual manufacturer in the CPR system, her share of the collective collection cost is $pC_i^{3P}$ where $p \in (0, 1)$. In general, a manufacturer will prefer CPR over IPR if there is a collection cost saving in the CPR system due to the system collector's economies of scale in collecting a large volume of used products. However, the CPR system has been criticized by manufacturers who are more efficient in collecting the used products due to fairness concerns and lack of incentives for environmental improvement (Atasu and Boyaci, 2010).

4.4.2. Producer vs. consumer pay

Some take-back laws (e.g., WEEE Directive) hold the producers financially responsible for the costs of the used product collection. Such a producer pays model has been considered in this paper. However, in practice, some take-back laws (e.g., California) hold the consumers financially responsible for their used product returns (consumer pays model). If a consumer pays model is implemented, then the manufacturer does not incur the collection cost and will be indifferent in the choice of reverse channel structures.

4.4.3. Producer-run vs. state-run collection

In this paper, we focus on the producer-run take-back system in which the manufacturer selects the reverse channel structure to achieve the collection target set by the take-back law. However, in practice, the state-run take-back system has been implemented in some countries such as China and Belgium and some states (e.g., Washington) in the U.S. In the state-run system, the governments or local municipalities undertake the used product collections and often charge a per unit collection and recovery fee (or tax) to the manufacturer. Model 3P generally applies to such a state-run system with some modifications to the collection cost. To illustrate, let $r$ be the per unit fee (tax) charged by the government or local municipality to the manufacturer. Then the manufacturer's total collection cost paid to the collector will be $nrQ$. The rest of model analysis is straightforward so we omit it.

5. Conclusions

In this paper we study CLSC models for a high-tech product which is featured with a short life-cycle and volatile demand. We focus on the manufacturer's choice of three alternative reverse channel structures (i.e., manufacturer collecting, retailer collecting, and third-party firm collecting) for collecting the used product from consumers. We characterize and compare the manufacturer's optimal production quantities and profits under the three alternative reverse channel structures. We also investigate the impacts of collection cost structures (i.e., economies and diseconomies of scale) and product take-back laws on the manufacturer's choice of reverse channel structures.

We provide some key insights on the manufacturer's choice of reverse channel structures in the CLSC for the high-tech product. We find that the manufacturer's choice of reverse channel structure is closely related to the collection methods deployed in the CLSC in two ways. First, if the collection cost structures of the manufacturer, retailer, and third-party firm are similar, then the reverse channel structure with retailer collecting is optimal to the manufacturer when the collection cost exhibits diseconomies of scale, whereas the reverse channel structure with manufacturer collecting is optimal to the manufacturer when the collection cost exhibits diseconomies of scale. Second, if the collection cost structures of the manufacturer, retailer, and third-party firm are different, then in general, the reverse channel structure with a lower marginal collection cost is optimal to the manufacturer under both economies and diseconomies of scale in collection cost.

Second, we provide insights on the impact of product take-back laws on the manufacturer's forward channel decision and overall profit in the CLSC. Our results suggest that if there are economies of scale in collection cost, then the legislator could set a relatively high collection rate target in the take-back law since a higher collection rate will not only improve environmental performance, but also increase the manufacturer's forward channel production quantity and overall profit, and make the high-tech product more available to end consumers. However, if there are diseconomies of scale in collection cost, then the legislator needs to realize that if the collection rate target is set too high in the take-back law, it may decrease the manufacturer's forward channel production quantity and overall profit, and make the high-tech product less available to end consumers. Our numerical results further show that take-back laws are more important to forcing the manufacturer to collect more used products to improve environmental performance when there are diseconomies of scale in the collection cost.

As a first step, in this paper we study the CLSC design for a high-tech product with a short life-cycle and uncertain demand. Future research should consider more complex models that relax some of our assumptions. For example, we use the commonly used single-period newsvendor model for the high-tech product CLSCs. Future research could consider multi-period stochastic inventory models when the high-tech product is relatively more durable. Also, due to demand uncertainty, to simplify our analysis, we assume the retail price of the original product is exogenous. Future research could study CLSC models with an endogenous retail price. Finally, we focus on two distinctive collection cost structures that exhibit economies and diseconomies of scale. In practice, collection cost structures may deviate from the ones we consider in this paper.

Appendix A. Proofs in Sections 2 and 3

Proof of Theorem 1. (i) If $C_i(\tau, Q) = C_i(\tau, Q)$, then after taking the first and second derivatives of $x^2(\tau)$ expressed in (2) with respect to $Q$, we get

$$d x^2(\tau)/dQ = p - c - (p - v)\tau, \quad d^2 x^2(\tau)/dQ^2 = -(p - v)\tau < 0$$

So $x^2(\tau)$ is concave in $Q$. Since $dx^2(0)/dQ = p - c - (\Delta - A)\tau > 0$ and by Assumption 3, $\lim_{\tau \to \infty} dx^2(\tau)/dQ = -(c - v) + (\Delta - A)\tau < 0$, there exists a unique optimal order quantity $Q^2 \in (0, \infty)$ that satisfies the first-order optimality condition (3).
(ii) Similarly, if $C_L(t, \tau) = C_D(t, \tau)$, then we have
\[ d^2 \pi^C(t)/d\tau^2 = -(p-v)f'(Q^C) - B\tau < 0. \]

Since $d\pi^C(0)/d\tau > 0$ and $\lim_{\tau \to -\infty} d\pi^C(\tau)/d\tau < 0$, there exists a unique optimal order quantity $Q^C \in (0, \infty)$ that satisfies the first-order optimality condition (4).

**Proof of Proposition 1.** (i) If $C_L(t, \tau) = C_D(t, \tau)$, then after plugging $Q^C$ and $Q^D$ into (29), we get $d\pi^C(Q^C)/d\tau = (\Delta - A)\tau > 0$, implying $Q^C > Q^D$. (ii) Similarly, if $C_L(t, \tau) = C_D(t, \tau)$, then after plugging $Q^D$ into (30), we get $d\pi^D(Q^D)/d\tau = \tau[(\Delta - A) - B\tau Q^D]$. If $\tau > \tau^D = (\Delta - A)/(BQ^D)$, then $d\pi^D(Q^D)/d\tau > 0$, implying $Q^D < Q^D$. □

**Proof of Proposition 2.** (i) If $C_L(t, \tau) = C_D(t, \tau)$, then after taking the first derivative of Eq. (3) with respect to $\tau$ and applying some algebraic manipulations, we get
\[ d\pi^C(\tau)/d\tau = (\Delta - A)/(p-v)f(Q^C) > 0. \]
(ii) Similarly, if $C_L(t, \tau) = C_D(t, \tau)$, then after taking the first derivative of Eq. (4) with respect to $\tau$ and applying some algebraic manipulations, we get
\[ d\pi^D(\tau)/d\tau = (\Delta - A - 2B\tau Q^D)/(p-v)f(Q^D) + B\tau. \]
We can verify that if $Q^C < Q^D$, the unique solution to $g(\tau) = 1$. We can verify that $d\pi^C(\tau)/d\tau < 0$ for all $\tau \in (0, \infty)$. Since $g(\tau) < 1$ for all $Q < Q^C$, $\pi^C(\tau)$ is concave and first increasing and then decreasing in $\tau$. Therefore, there must exist a unique and finite optimal order quantity $Q^C$ satisfying (10). □

**Proof of Theorem 2.** (i) If $C_L(t, \tau) = C_D(t, \tau)$, then after taking the first and second derivatives of $\pi^C(\tau)$ with respect to $\tau$, we get
\[ d\pi^C(\tau)/d\tau = (\Delta - A)/(p-v)[1 - F(Q^C)][1 - g(Q^C)] - [(c - v - (\Delta - A)]r, \]
\[ d^2 \pi^C(\tau)/d\tau^2 = -(p-v)f'(Q^C)[1 - g(Q^C)] - (p-v)(1 - F(Q^C))g'(Q^C). \]
If $\lim_{\tau \to -\infty} g(Q^C) < 1$, then from (34), we have $d^2 \pi^C(\tau)/d\tau^2 < 0$, implying $\pi^C(\tau)$ is concave in $Q$. Similarly, since $d\pi^C(\tau)/d\tau > 0$ and $\lim_{\tau \to -\infty} d\pi^C(\tau)/d\tau < 0$, there must exists a finite optimal order quantity $Q^C$ satisfying the first-order optimality condition (10). If $\lim_{\tau \to -\infty} g(Q^C) > 1$, then we have $d\pi^C(\tau)/d\tau < 0$ for all $\tau \in (0, \infty)$ and $\pi^C(\tau)$ is concave and first increasing and then decreasing in $\tau$. Therefore, there must exist a unique and finite optimal order quantity $Q^C$ satisfying (10). □

**Proof of Proposition 3.** (i) If $C_L(t, \tau) = C_D(t, \tau)$, then after plugging $Q^M$ expressed in (12) into (31), we get $d\pi^D(\tau)/d\tau > 0$, implying $Q^D > Q^M$. (ii) Similarly, if $C_L(t, \tau) = C_D(t, \tau)$, then after plugging $Q^M$ into (33), we get $d\pi^D(\tau)/d\tau = \tau[(\Delta - A) - B\tau Q^M]$. If $\tau > \tau^M$, then $d\pi^D(\tau)/d\tau > 0$, implying $Q^M > Q^M$. □

**Proof of Proposition 4.** (i) If $C_L(t, \tau) = C_D(t, \tau)$, then after taking the first derivative of Eq. (10) with respect to $\tau$ and applying some algebraic manipulations, we get
\[ d\pi^C(\tau)/d\tau = \frac{\Delta - A}{(p-v)[f(Q^C)[1 - g(Q^C)] + [1 - F(Q^C)]g'(Q^C)]} > 0. \]
(ii) Similarly, if $C_L(t, \tau) = C_D(t, \tau)$, then after taking the first derivative of Eq. (10) with respect to $\tau$ and applying some algebraic manipulations, we get
\[ d\pi^C(\tau)/d\tau = \frac{\Delta - A}{(p-v)[f(Q^C)[1 - g(Q^C)] + [1 - F(Q^C)]g'(Q^C)]} > 0. \]
Let \( \pi \) be the unique and \( Q = C_{\pi}(r, Q) \) then we can verify that
\[
\pi = \frac{\Delta - A - 4B r Q^R}{(p-v)[1-g(Q^M)]+[1-f(Q^M)]E'g(Q^M)]+2B r^2
\]
We can verify if \( Q^R < Q^M = (\Delta - A)/4Br \), then \( dQ^R/dr > 0 \), otherwise, \( dQ^R/dr < 0 \). □

**Proof of Theorem 4.** The proof is similar to the proof of Theorem 2 so we omit it. □

**Proof of Theorem 5.** (i) If \( C_{\pi}(r, Q) = C_{\pi}(r, Q) \), then we see that the first-order optimality conditions (10) in Model M and (20) in Model R are exactly the same. Therefore, \( Q^M = Q^R = Q^P \). (ii) If \( C_{\pi}(r, Q) = C_{\pi}(r, Q) \), then after plugging \( Q^P \) into (31), we get \( \Delta M(Q^R)/dQ = Br Q^R > 0 \), implying \( Q^R < Q^M = Q^P \). □

**Proof of Theorem 6.** (i) If \( C_{\pi}(r, Q) = C_{\pi}(r, Q) \), then \( Q^M = Q^R = Q^P \) by Theorem 5(i). From the manufacturer's profit functions (9) and (19) in Model M and Model R, we have \( \pi^M(Q^M) - \pi^M(Q^R) = - (1/2)B r^2 < 0 \), implying \( \pi^M(Q^M) < \pi^M(Q^R) \). (ii) If \( C_{\pi}(r, Q) = C_{\pi}(r, Q) \), then we have \( \pi^M(Q^M) - \pi^M(Q^R) = \pi^M(Q^M) - \pi^M(Q^R) + (1/2)B r Q^R > 0 \), implying \( \pi^M(Q^M) > \pi^M(Q^R) \). In both cases, it follows from (26) that \( \pi^M(Q^P) \leq \pi^M(Q^M) \). □

### Appendix B. Decentralized CLSC models in Section 4.2

In this appendix, we provide our analyses of the three decentralized CLSC models based upon the collection cost function defined by Assumption 5. To be concise, we omit the descriptions and timings of events of the CLSC models as they are similar to those models in Section 2.

**Model M: closed-loop supply chain with manufacturer collecting**

Similar to Model M in Section 2.2, the manufacturer's expected profit function \( \pi^M(Q) \) as a function of the retailer's order quantity \( Q \) is given by
\[
\pi^M(Q) = \frac{[p - (p-v)[1-F(Q^M)][1-g(Q^M)] - (c-v)}{\text{manufacturer's forward channel profit}} + \frac{\int_0^Q [\Delta r x - C_{\pi}(r, x)]f(x) dx + \int_0^\infty [\Delta r Q - C_{\pi}(r, x)]f(x) dx}{\text{manufacturer's reverse channel profit}}
\]

**Theorem B1.** There exists a unique and finite optimal order quantity \( Q^M \) that maximizes the manufacturer's expected profit in Model M:

(i) If \( C_{\pi}(r, s) = C_{\pi}(r, s) \), then \( Q^M \) satisfies the following first-order optimality condition:
\[
(p-v)[1-F(Q^M)](1-g(Q^M)] - (c-v) + \frac{\Delta A r [1-F(Q^M)]]}{\text{manufacturer's forward channel economic trade-off}} = 0.
\]

(ii) If \( C_{\pi}(r, s) = C_{\pi}(r, s) \), then \( Q^M \) satisfies the following first-order optimality condition:
\[
(p-v)[1-F(Q^M)][1-g(Q^M)] - (c-v) + \frac{[\Delta A r - B r Q^M] - (c-v)}{\text{manufacturer's forward channel economic trade-off}} = 0.
\]

**Proof of Theorem B1.** (i) If \( C_{\pi}(r, s) = C_{\pi}(r, s) \), then after taking the first derivative of \( \pi^M(Q) \) expressed in (B1) with respect to \( Q \), we get
\[
d\pi^M(Q)/dQ = [1-F(Q)](p-v)[1-g(Q)] + (\Delta A r) - (c-v).
\]
If \( \lim_{Q \to 0} g(Q) \leq 1 + \Delta A r / (p-v) \), \( \pi^M(Q) \) is decreasing in \( Q \). Since \( d\pi^M(Q)/dQ > 0 \) and \( \lim_{Q \to \infty} d\pi^M(Q)/dQ < 0 \), there is a unique \( Q^M \) satisfying (B2). If \( \lim_{Q \to \infty} g(Q) > 1 + \Delta A r / (p-v) \), then let \( Q^P \) be the unique and finite solution to \( (p-v)[1-g(Q)] + (\Delta A r) = 0 \). We can verify that \( d\pi^M(Q)/dQ > 0 \) for all \( Q \in [Q^P, \infty) \) and \( \pi^M(Q) \) is concave and first increasing and then decreasing under \( Q \in [0, Q^P] \). Hence, \( \pi^M(Q) \) must be uni-modal in \( Q \in [0, Q^P] \). Therefore, there must exist a unique and finite \( Q^M \) satisfying (B2).

(ii) If \( C_{\pi}(r, s) = C_{\pi}(r, s) \), then after taking the first derivative of \( \pi^M(Q) \) with respect to \( Q \), we get
\[
d\pi^M(Q)/dQ = [1-F(Q)](p-v)[1-g(Q)] + (\Delta A r) - B r Q^R - (c-v).
\]
Let \( k(Q) = (p-v)g(Q) + (\Delta A r) - B r Q^R \). Then \( k(Q) \) is decreasing in \( Q \) and \( \lim_{Q \to 0} k(Q) = 0 \) and \( \lim_{Q \to \infty} k(Q) = 0 \). So there must exist a unique and finite \( Q^M \) that satisfies \( k(Q^M) = 0 \). It is easy to verify that \( \pi^M(Q) \) is concave and first increasing and then decreasing under \( Q \in [0, Q^P] \) and strictly decreasing in \( Q \in [Q^P, \infty) \). Therefore, there must exist a unique and finite \( Q^M \) satisfying (B3). □
Model R: closed-loop supply chain with retailer collecting

In the manufacturer–leader–retailer–follower game of Model R, we work backward starting with the retailer’s expected profit function \( \hat{\pi}(Q) \) given by

\[
\hat{\pi}(Q) = \int_0^Q [p x + v (Q - x)] f(x) \, dx + \int_Q^\infty [p Q f(x) \, dx - w Q + \int_0^Q [b r x - \mu r (r, x)] f(x) \, dx + \int_Q^\infty [b r Q - \mu r (r, Q)] f(x) \, dx.
\]

Lemma B1. For a given wholesale price and buyback price \( (w, b) \), there exists a unique optimal buyback price \( w^*(w, b) \) that maximizes the retailer’s expected profit in Model R:

\[
\pi^*(w, b) \quad \text{if } \quad C_l(r, s) = C_{\pi}(r, s),
\]

\[
\frac{\partial \pi^*}{\partial (w, b)} \quad \text{if } \quad C_l(r, s) = C_{\pi}(r, s).
\]

Proof of Lemma B1. The proof is similar to the proof of Lemma 1 so we omit it. \( \square \)

Similarly, since for any fixed \( b \), there exists a one-to-one relationship between \( w^*(w, b) \) and \( w \), we work with the manufacturer’s inverse function of \( w^*(w, b) \) given by

\[
w^*(Q, b) = \begin{cases} 
(p - (p - v) f(Q) + (b - A) \tau [1 - F(Q)]) & \text{if } \ C_l(r, s) = C_{\pi}(r, s), \\
(p - (p - v) f(Q) + [(b - A) \tau - B r^2 Q] [1 - F(Q)]) & \text{if } \ C_l(r, s) = C_{\pi}(r, s).
\end{cases}
\]

Then we can express the manufacturer’s expected profit function as follows:

\[
\hat{M}(Q) = \begin{cases} 
(p - (p - v) f(Q) + (\Delta - A) \tau [1 - F(Q)] - c) Q + \int_0^Q (\Delta - b) x f(x) \, dx & \text{if } \ C_l(r, s) = C_{\pi}(r, s), \\
(p - (p - v) f(Q) + [(\Delta - A) \tau - B r^2 Q] [1 - F(Q)] - c) Q + \int_0^Q (\Delta - b) x f(x) \, dx & \text{if } \ C_l(r, s) = C_{\pi}(r, s).
\end{cases}
\]

Theorem B2. There exists a unique and finite optimal order quantity \( Q^* \) that maximizes the manufacturer’s expected profit in Model R:

\[
\begin{align*}
\text{(i) If } C_l(r, Q) = C_{\pi}(r, Q), & \quad \text{then } Q^* \text{ satisfies the following first-order optimality condition:} \\
(p - v)[1 - F(Q^*)][1 - g(Q^*)] - (c - v) & + (\Delta - A) \tau [1 - F(Q^*)] = 0, \\
\text{manufacturer’s forward channel trade–off} & \text{ manufacturer’s reverse channel trade–off} \\
\text{(ii) If } C_l(r, Q) = C_{\pi}(r, Q), & \quad \text{then } Q^* \text{ satisfies the following first-order optimality condition:} \\
(p - v)[1 - F(Q^*)][1 - g(Q^*)] - (c - v) + (\Delta - A) \tau [1 - F(Q^*)] & = 0, \\
\text{manufacturer’s forward channel trade–off} & \text{ manufacturer’s reverse channel trade–off}.
\end{align*}
\]

Proof of Theorem B2. (i) If \( C_l(r, s) = C_{\pi}(r, s) \), then it is easy to verify that \( \frac{\partial \hat{M}(Q, b)}{\partial b} < 0 \). Since \( \hat{M}(Q, b) \) is strictly decreasing in \( b \geq A \), the optimal buyback price \( b^* \) must satisfy \( b^* = A \). After plugging \( b = A \) into the partial derivative of \( \hat{M}(Q, b) \) with respect to \( Q \), we get

\[
\frac{\partial \hat{M}(Q, A)}{\partial Q} = (p - v)[1 - F(Q)][1 - g(Q)] - (c - v) + (\Delta - A) \tau [1 - F(Q)].
\]

It follows from Proof of Theorem B1(i) that there is a unique \( Q^* \in (0, \infty) \) satisfying (B9).

(ii) Similarly, if \( C_l(r, s) = C_{\pi}(r, s) \), then \( \hat{M}(Q, b) \) is decreasing in \( b \geq A \). So \( b^* \) must satisfy \( b^* = A \). After plugging \( b = A \) into \( w^*(Q, b) \) expressed in (B7), we get

\[
w^*(Q, A) = (p - v - B r^2 Q)[1 - F(Q)] + v.
\]

Since \( w^*(Q, A) > c \) by Assumption 1, it requires \( (p - v - B r^2 Q)[1 - F(Q)] > c - v \). Therefore, to be feasible, the retailer’s order quantity \( Q \) must satisfy \( Q < Q_{\max} = (p - v)/(B r^2) \).

After plugging \( b = b^* = A \) into the partial derivative of \( \hat{M}(Q, b) \) with respect to \( Q \), we get

\[
\frac{\partial \hat{M}(Q, A)}{\partial Q} = (p - v)[1 - F(Q)][p - v - B r^2 Q][1 - g(Q)] - B r^2 Q([1 - (c - v) + (\Delta - A) \tau [1 - F(Q)])].
\]

Note that the term, \( -(c - v) + (\Delta - A) \tau [1 - F(Q)] < -(c - v) + (\Delta - A) \tau < 0 \) by Assumption 3. Define \( h(Q) = (p - v - B r^2 Q)[1 - g(Q)] - B r^2 Q \). If \( g(Q_{\max}) \leq 1 \), then \( h(Q) \) is decreasing in \( Q \). Since \( h(0) > 0 \) and \( h(Q_{\max}) < 0 \), there must exist a unique \( Q^* \in (0, Q_{\max}) \) that satisfies
(ii) If $C_1(r,s) = C_0(r,s)$, then $Q^M = Q^R$ by Theorem 7(i). Since $\pi^M(Q^M) - \pi^R(Q^R) = -B^2r^2/2 < 0$, we have $\hat{x}^M(Q^M) - \hat{x}^R(Q^R) = \hat{x}^M(Q^R)$. It follows from (B11) that $\pi^M(Q^R) > \pi^R(Q^R)$. □

Appendix C. Decentralized CLSC models in Section 4.3

In this appendix, we provide our analyses of the CLSC models based upon the collection cost function defined by Assumption 6. To be concise, we omit the descriptions and timings of events of the CLSC models as they are similar to those models in Section 2.

After replacing A and B with $A'$ and $B'$ in Model $i = [M, R, 3P]$ in Theorems 1–4, we have the following Theorems C1–C3 that characterize the optimal order quantity $Q^i$ in Model $i$. All Proofs of Theorems C1–C3 follow directly from the proofs of Theorems 2–4, so we omit them.

Theorem C1. There exists a unique and finite optimal order quantity $Q^M$ that maximizes the manufacturer's expected profit in Model M:

(i) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

(ii) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

Theorem C2. There exists a unique and finite optimal order quantity $Q^R$ that maximizes the manufacturer's expected profit in Model R:

(i) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

(ii) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

Theorem C3. There exists a unique and finite optimal order quantity $Q^{3P}$ that maximizes the manufacturer's expected profit in Model 3P:

(i) If $C_1^{3P}(r, Q) = C_0^{3P}(r, Q)$, then $Q^{3P}$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^{3P})) - (c - v) + \frac{(\Delta - A^{3P})}{r} = 0.
\]  

Appendix C. Decentralized CLSC models in Section 4.3

In this appendix, we provide our analyses of the CLSC models based upon the collection cost function defined by Assumption 6. To be concise, we omit the descriptions and timings of events of the CLSC models as they are similar to those models in Section 2.

After replacing A and B with $A'$ and $B'$ in Model $i = [M, R, 3P]$ in Theorems 1–4, we have the following Theorems C1–C3 that characterize the optimal order quantity $Q^i$ in Model $i$. All Proofs of Theorems C1–C3 follow directly from the proofs of Theorems 2–4, so we omit them.

Theorem C1. There exists a unique and finite optimal order quantity $Q^M$ that maximizes the manufacturer’s expected profit in Model M:

(i) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

(ii) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

Theorem C2. There exists a unique and finite optimal order quantity $Q^R$ that maximizes the manufacturer’s expected profit in Model R:

(i) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

(ii) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

Theorem C3. There exists a unique and finite optimal order quantity $Q^{3P}$ that maximizes the manufacturer’s expected profit in Model 3P:

(i) If $C_1^{3P}(r, Q) = C_0^{3P}(r, Q)$, then $Q^{3P}$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^{3P})) - (c - v) + \frac{(\Delta - A^{3P})}{r} = 0.
\]  

Appendix C. Decentralized CLSC models in Section 4.3

In this appendix, we provide our analyses of the CLSC models based upon the collection cost function defined by Assumption 6. To be concise, we omit the descriptions and timings of events of the CLSC models as they are similar to those models in Section 2.

After replacing A and B with $A'$ and $B'$ in Model $i = [M, R, 3P]$ in Theorems 1–4, we have the following Theorems C1–C3 that characterize the optimal order quantity $Q^i$ in Model $i$. All Proofs of Theorems C1–C3 follow directly from the proofs of Theorems 2–4, so we omit them.

Theorem C1. There exists a unique and finite optimal order quantity $Q^M$ that maximizes the manufacturer’s expected profit in Model M:

(i) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

(ii) If $C_1^M(r, Q) = C_0^M(r, Q)$, then $Q^M$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^M)) - (c - v) + \frac{(\Delta - A^M)}{r} = 0.
\]  

Theorem C2. There exists a unique and finite optimal order quantity $Q^R$ that maximizes the manufacturer’s expected profit in Model R:

(i) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

(ii) If $C_1^R(r, Q) = C_0^R(r, Q)$, then $Q^R$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^R)) - (c - v) + \frac{(\Delta - A^R)}{r} = 0.
\]  

Theorem C3. There exists a unique and finite optimal order quantity $Q^{3P}$ that maximizes the manufacturer’s expected profit in Model 3P:

(i) If $C_1^{3P}(r, Q) = C_0^{3P}(r, Q)$, then $Q^{3P}$ satisfies the following first-order optimality condition:

\[
(p - v)(1 - F(Q^{3P})) - (c - v) + \frac{(\Delta - A^{3P})}{r} = 0.
\]
(ii) If $C^F_j(r, Q) = C^D_j(r, Q)$, then $Q^D$ satisfies the following first-order optimality condition:

$$\left(\rho - v\right)\left[1 - F(Q^D)\right] \left[1 - g(Q^D)\right] - (c - v) + \left(\Delta - A^D\right)\tau - B^D\tau Q^D \neq 0.$$  

Proof of Theorem 9. (i) If $C^F_j(r, Q) = C^D_j(r, Q)$, then it is easy to verify that $\partial Q^F/\partial \delta^F < 0$.

(ii) If $C^F_j(r, Q) = C^D_j(r, Q)$, then from (C2), (C4) and (C6) that the optimal order quantities $Q^F$ and $Q^D$ in Model i and Model j can be expressed as

$$(p - v)\left[1 - F(Q^F)\right] \left[1 - g(Q^F)\right] - (c - v) + \Delta \tau - MC^F_j(r, Q^F) = 0 \quad \text{and}$$

$$\left(p - v\right)\left[1 - F(Q^D)\right] \left[1 - g(Q^D)\right] - (c - v) + \Delta \tau - MC^D_j(r, Q^D) = 0.$$  

After plugging $Q^F$ into $(\partial Q^F/\partial Q)$, we get

$$\frac{\partial Q^F}{\partial Q} = \left(p - v\right)\left[1 - F(Q^F)\right] \left[1 - g(Q^F)\right] - (c - v) + \Delta \tau - MC^F_j(r, Q^F) = MC^D_j(r, Q^F) - MC^F_j(r, Q^F).$$

If $MC^F_j(r, Q^F) > MC^D_j(r, Q^D)$, then $(\partial Q^F/\partial Q) > 0$, implying $Q^F > Q^D$. Similarly, we can prove if $MC^F_j(r, Q^F) < MC^D_j(r, Q^D)$, then $Q^F < Q^D$. □

Proof of Theorem 10. (i) If $C^F_j(r, Q) = C^D_j(r, Q)$, then the manufacturer's optimal expected profits in Model M and Model R can be expressed as

$$\begin{align*}
\hat{x}^M(Q^M) &= \left(p - c - (p - v) F(Q^M) + (\Delta - A^M)\tau\right) Q^M - B^M\tau^2/2 \quad \text{and} \\
\hat{x}^R(Q^R) &= \left(p - c - (p - v) F(Q^R) + (\Delta - A^R)\tau\right) Q^R.
\end{align*}$$

After applying some algebraic manipulations, we get

$$\begin{align*}
\hat{x}^M(Q^M) &= \hat{x}^M(Q^M) - \hat{x}^R(Q^R) + c^M(r, Q^R) - C^M_j(r, Q^R) - B^M\tau^2/2. \\
\hat{x}^R(Q^R) &= \hat{x}^R(Q^R) + c^F_j(r, Q^F) - C^F_j(r, Q^F).
\end{align*}$$

(ii) Similarly, if $C^F_j(r, Q) = C^R_j(r, Q)$, then the manufacturer's optimal expected profits in Model M and Model R can be expressed as

$$\begin{align*}
\hat{x}^M(Q^M) &= \left(p - c - (p - v) F(Q^M) + (\Delta - A^M)\tau\right) Q^M - B^M\tau^2/2 \quad \text{and} \\
\hat{x}^R(Q^R) &= \left(p - c - (p - v) F(Q^R) + (\Delta - A^R)\tau\right) Q^R - B^R\tau^2/2.
\end{align*}$$

After applying some algebraic manipulations, we get

$$\begin{align*}
\hat{x}^M(Q^M) &= \hat{x}^M(Q^M) - \hat{x}^R(Q^R) + c^M_j(r, Q^R) - C^M_j(r, Q^R) + B^M\tau(Q^M)^2/2. \\
\hat{x}^R(Q^R) &= \hat{x}^R(Q^R) + c^F_j(r, Q^F) - C^F_j(r, Q^F).
\end{align*}$$

(iii) It follows from (26) that the manufacturer's expected profit in Model 3P can be expressed as

$$\begin{align*}
\hat{x}^M(Q^M) &= \left(p - c - (p - v) F(Q^M)\right) Q^M + A^M\tau^3 - C^M_j(r, Q^M) - \tau^3\tau
\end{align*}$$

Then we have

$$\hat{x}^M(Q^M) = \hat{x}^F(Q^M) - \hat{x}^D(Q^D) + C^F_j(r, Q^F) - C^D_j(r, Q^D) - \tau^3\tau.$$  

Since $\hat{x}^M(Q^M) > \hat{x}^D(Q^D)$, if $C^F_j(r, Q^F) - C^D_j(r, Q^D) > B^F(\tau^3)^2/2$, then $\hat{x}^M(Q^M) - \hat{x}^D(Q^D) > 0$.

References


