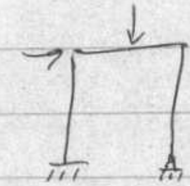
Superposition of Mechanisms

If N crit moments + R redundancies

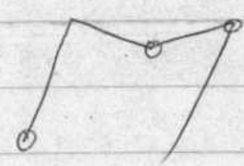
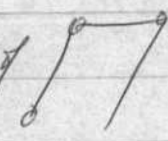
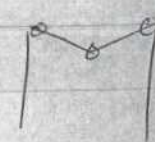
$\exists$ $N-R$ indep equil eqns

an equil eqn assoc (via virt wk) with each mech.

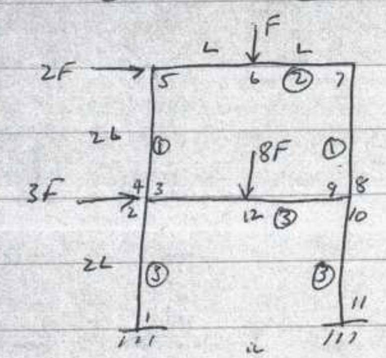
$\therefore$ also $N-R$ elem mech.

note in  $R=2$ $N=4$ 2 eq eqns

3 mech

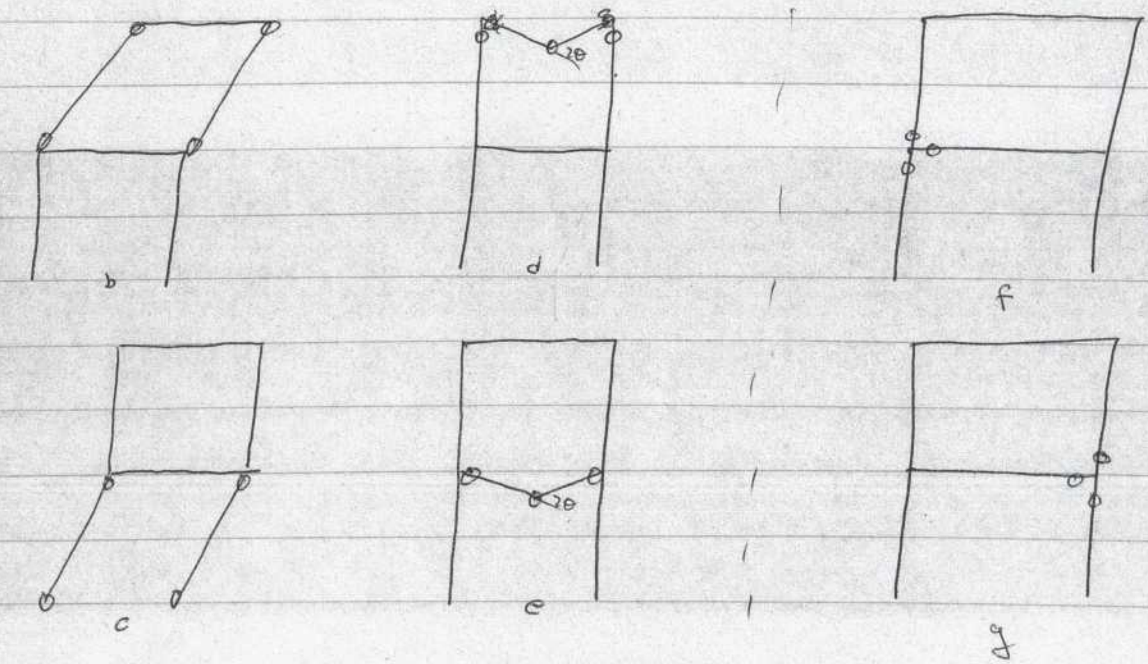
but  a combination of  and 

& if a comparison is made, the associated equil eqns are likewise consider

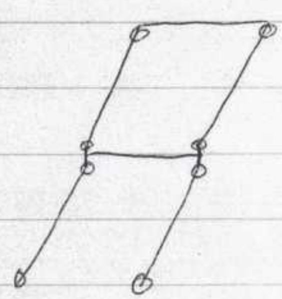


$N = 12 \qquad R = 6 \qquad N - R = 6$

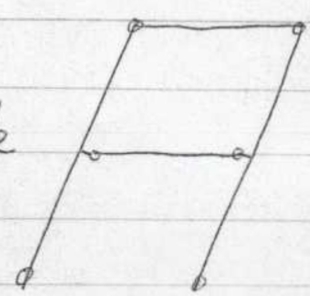
elem mech



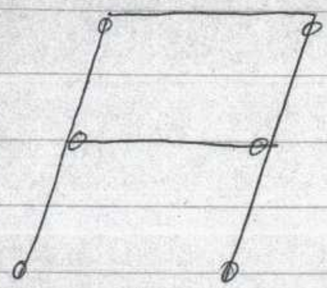
now consider a combination of b + c

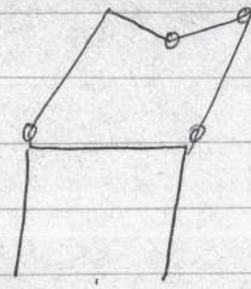


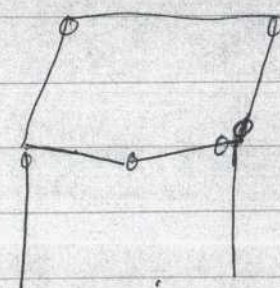
which is more likely to be



∴ put in as elem mech's f + g


h


i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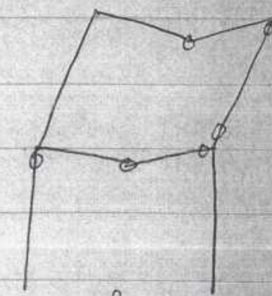
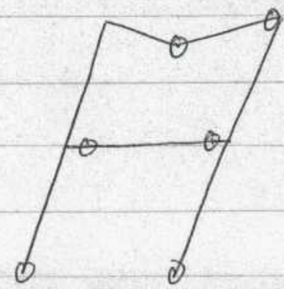
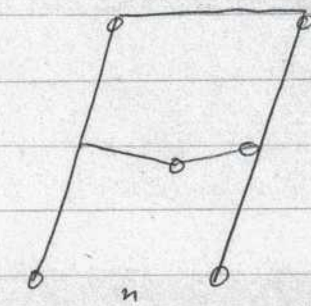
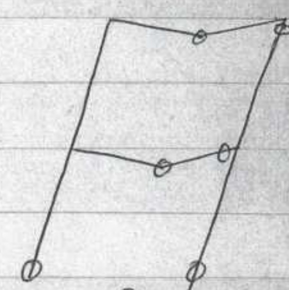

k


l


m


n


o

note for f

$$\mathcal{H}_i = \theta(M_2 - M_3 - M_4) \qquad \mathcal{H}_e = 0$$

$\therefore$ <u>not</u> KAF

for upper bds

mech (b) $FL\theta(2\cdot 2) = M_0\theta(1+1+1+1)$ $f^+ = 1$

(c) $FL\theta(3\cdot 2 + 2\cdot 2) = M_0\theta(3+3+3+3)$ $f^+ = \frac{12}{10} = 1.2$

(d) $FL\theta(1) = M_0\theta(1+2\cdot 2+1)$ $f^+ = 6$

(e) $FL\theta(8) = M_0\theta(3+2\cdot 3+3)$ $f^+ = \frac{12}{8} = 1.5$

now try combined panel mechs

(h): $FL\theta(2\cdot 4+3\cdot 2) = M_0\theta(1+1+3+3+3+3)$ $f^+ = \frac{14}{14} = 1$

or, could take $(b) + (c) - \overset{\text{swings}}{(f)} - \overset{\text{swings}}{(g)}$

$$R.S. = W_i = M_0\theta(4+12-1-1) = 14M_0\theta$$

$$W_e = FL\theta(4+10) = 14M_0\theta$$

note in combining, must seek to combine mechs in such a way as to eliminate hinges + thus reduce W_i

note we will otherwise get a weighted mean

Try a panel + add a beam mech

b + h are best panel , e is best beam

try b + e and h + e

$\downarrow$

$j: \; FL\theta(2\cdot2 + 8) = M_0\theta(1+1+1+3+\cancel{6}+3) \qquad f = 1.25$

$h+e: \; n: \; FL\theta(\cancel{2}\times4 + 3\times2 + 8\times1)^{22} = M_0\theta(1+1+\cancel{6}+\cancel{6}+3+3)^{20} \qquad f = \frac{20}{22} = \frac{10}{11} = .909$

n is lowest, work from it:

n + other beam ie n + d : o

$o: \; \overset{M_0}{\cancel{FL}}\theta(2\cdot\cancel{2} + \cancel{2} + 3\cdot2 + 3\cdot2 + 3+3)^{24} = \overset{FL}{\cancel{M_0}}\theta(1+8+4\cdot2+3\cdot2)^{23} \qquad f = \frac{24}{23}$

settle for .909

although 4 ~~?~~ to go, unreasonable to continue

i =	$FL\theta(4+1)$ =	$M_0\theta(4+6-2)$	$f = 1.6$
k	12 + 1	15 + 6 − 2	1.46
l	10 + 8	12 + 12 + 1 − 4	1.056
m	14 + 1	14 + 6 − 2	1.2

To find lower bounds: use virtual work

(b) : $-M_4 + \underline{M_5} - \underline{M_7} + M_8 = 4FL$ ✓

c $-\underline{M_1} + M_2 - M_{10} + \underline{M_{11}} = 10FL$ ✓

d $-\underline{M_5} + 2M_6 - \underline{M_7} = FL$ ✓

e $-M_3 + 2M_{12} - \underline{M_9} = 8FL$

f $M_2 - M_3 - M_4 = 0$

g $M_8 + \underline{M_9} - M_{10} = 0$ ✓

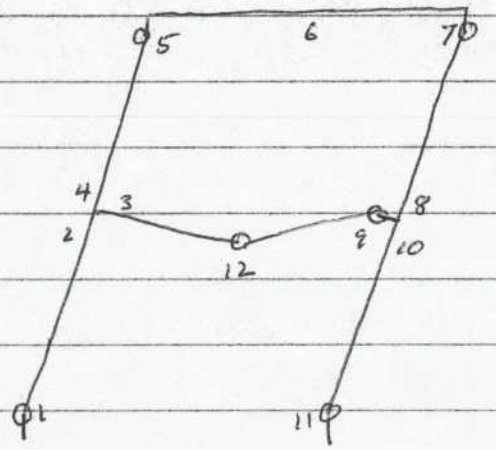
subst FL + M's known from n + let $m_{10} = M_{10}/m_0$ etc

+ calc rest

see next page

$m_1 = -3$	$m_5 = 1$	$m_9 = -3$
$m_2 = m_{10} + \frac{34}{11}$	$m_6 = \frac{5}{11}$	$m_{10} = m_{10}$
$m_3 = \frac{19}{11}$	$m_7 = -1$	$m_{11} = 3$
$m_4 = m_{10} + \frac{15}{11}$	$m_8 = 3 + m_{10}$	$m_{12} = 3$

(n):



$\underline{m_1 = -3}$ $\quad$ $\underline{m_9 = -3}$

$\underline{m_5 = 1}$ $\quad$ $\underline{m_{11} = 3}$

$\underline{m_7 = -1}$ $\quad$ $\underline{m_{12} = 3}$

$f = 10/11$

(e) : $m_3 = -80/11 + 6 + 3$ $\qquad$ $\underline{m_3 = 19/11}$ OK

(d) : $-1 + 2m_6 + 1 = 10/11$ $\qquad$ $\underline{m_6 = 5/11}$ OK

note - define in terms of m_{10}

(g) : $\underline{m_8 = m_{10} + 3}$

(c) : $\underline{m_2} = 100/11 + 3 + m_{10} - 3 = \underline{m_{10} + 34/11}$

(b) : $m_4 = -40/11 + 1 + 1 + m_{10} + 3 = \underline{m_{10} + 15/11}$

(f) : $\cancel{m_{10}} + \frac{34}{11} - \frac{19}{11} - \cancel{m_{10}} - \frac{15}{11} = 0$ ✓

now, $m_1, m_3, m_5, m_6, m_7, m_9, m_{11}, m_{12}$ are SA

we must also satisfy the SA of m_2, m_4, m_8, m_{10}

$-3 \le m_2 \le 3 \rightarrow -3 \le m_{10} + 34/11 \le 3$ i

$-1 \le m_4 \le 1 \rightarrow -1 \le m_{10} + 15/11 \le 1$ ii

$-1 \le m_8 \le 1 \quad -1 \le m_{10} + 3 \le 1$ iii

$-3 \le m_{10} \le 3$ iv

i $\quad -33 \le 11m_{10} + 34 \le 33$
$-67 \le 11 m_{10} \le 1$

ii $\quad -11 \le 11m_{10} + 15 \le 11$
$\underline{\underline{-26}} \le 11 m_{10} \le -4$

iii $\quad -4 \le m_{10} \le -2$
$-44 \le 11m_{10} \le \underline{\underline{-22}}$

iv $\quad -33 \le 11 m_{10} \le 33$

notice that $-26 \le 11 m_{10} \le -22$ satisfies all

$\rightarrow -\frac{26}{11} \le m_{10} \le -2$ since there is at least one OK SA

$f = 10/11$ is also a L.B. $\therefore f_0 = 10/11$