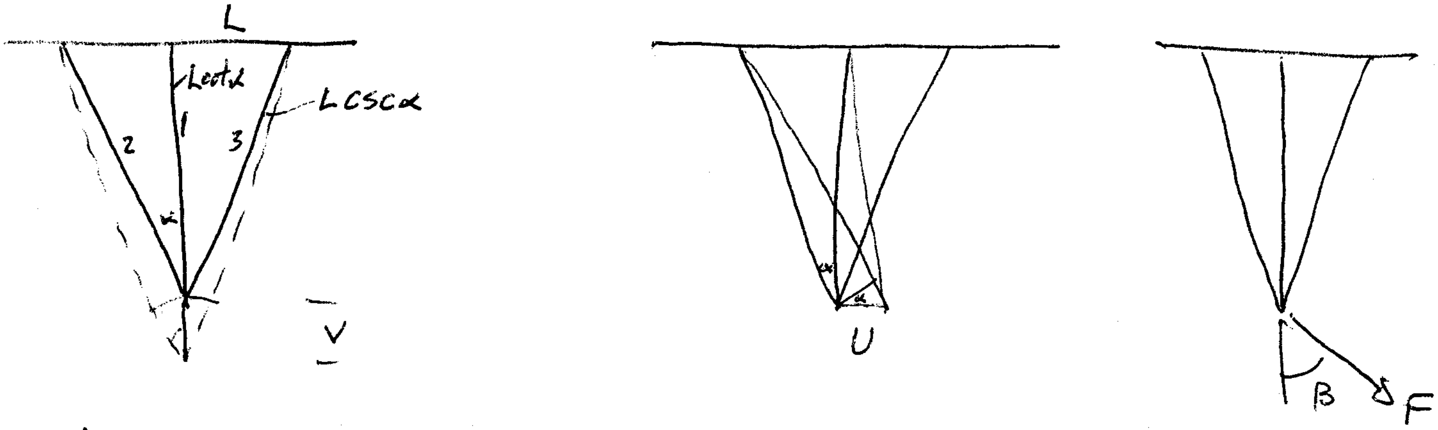
Notes – 3 bar truss, oblique load



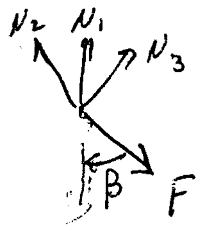
strain/displacement

$$\epsilon_1 = \frac{V}{L\cot\alpha} = \frac{V}{L}\tan\alpha$$

$$\epsilon_2 = \frac{V\cos\alpha}{L\csc\alpha} + \frac{U\sin\alpha}{L\csc\alpha} = \frac{V}{L}\sin\alpha\cos\alpha + \frac{U}{L}\sin^2\alpha$$

$$\epsilon_3 = \quad = \frac{V}{L}\sin\alpha\cos\alpha - \frac{U}{L}\sin^2\alpha$$

equil:



$$F\sin\beta = (N_2 - N_3)\sin\alpha$$

$$F\cos\beta = N_1 + (N_2 + N_3)\cos\alpha$$

$$N_i = \sigma_i A$$

if elastic $\quad N_i = AE\epsilon_i$

$$N_1 = \frac{AE}{L}V\tan\alpha$$

$$N_2 = \frac{AE}{L}\left(V\sin\alpha\cos\alpha + U\sin^2\alpha\right)$$

$$N_3 = \frac{AE}{L}\left(\quad '' \quad - \quad '' \quad \right)$$

subst into equil

$$F \sin\beta = 2\frac{AEU}{L}\sin^3\alpha$$

$$F\cos\beta = \frac{AEV}{L}\tan\alpha + 2\frac{AEV}{L}\sin\alpha\cos^2\alpha$$

$$\frac{AEU}{L} = \frac{F\sin\beta}{2\sin^3\alpha}$$

$$\frac{AEV}{L} = \frac{F\cos\beta}{\tan\alpha + 2\sin\alpha\cos^2\alpha}$$

$$N_1 = \frac{F\cos\beta}{1+2\cos^3\alpha} \qquad > 0 \text{ if } \quad \beta < \frac{\pi}{2}$$

$$N_2 = \overset{>0 \;\beta<\frac{\pi}{2}}{\frac{F\cos\beta\,\sin\alpha\cos^2\alpha}{\cancel{\tan\alpha} + 2\sin\alpha\cos^3\alpha}} + \overset{>0}{\frac{F\sin\beta}{2\sin\alpha}}$$

$$= \frac{F\cos\beta\,\cos^2\alpha}{1+2\cos^3\alpha} + \frac{F\sin\beta}{2\sin\alpha} > 0$$

$$N_3 = \quad '' \quad - \quad ''$$

clearly $N_3 < N_2$ and $|N_3| < |N_2|$

$\therefore$ N_3 will <u>not</u> yield first

(a) if N_1 yields first: $N_1 = +N_0$

and $F_{ce}^{(a)} = N_0 \dfrac{(1+2\cos^3\alpha)}{\cos\beta}$

then $N_2^{(a)} = \dfrac{N_0\cancel{(1+2\cos^3\alpha)}\cos^2\alpha}{\cancel{(1+2\cos^3\alpha)}} + \dfrac{N_0\tan\beta(1+2\cos^3\alpha)}{2\sin\alpha}$

since $N_2 > 0$

need only show $N_2 < N_0$

or

$$\cos^2\alpha + \tan\beta\frac{(1+2\cos^3\alpha)}{2\sin\alpha} \overset{?}{<} 1$$

all terms > 0

$$2\sin\alpha\cos^2\alpha + \tan\beta(1+2\cos^3\alpha) \quad | \quad 2\sin\alpha$$

$$\tan\beta(1+2\cos^3\alpha) \quad | \quad 2\sin\alpha(1-\cos^2\alpha)$$

$$\tan\beta \quad | \quad \frac{2\sin^3\alpha}{1+2\cos^3\alpha}$$

$\therefore$ $N_2 < N_0$ if $\tan\beta < \dfrac{2\sin^3\alpha}{1+2\cos^3\alpha}$

$\therefore$ bar 1 will yield first if $\tan\beta < \dfrac{2\sin^3\alpha}{1+2\cos^3\alpha}$

b) if N_2 yields first $N_2 = +N_0$

$$F_e^{(b)}\left[\cos\beta\,\frac{\cos^2\alpha}{1+2\cos^3\alpha} + \frac{\sin\beta}{2\sin\alpha}\right] = N_0$$

$$\left(\frac{2\sin\alpha\cos\beta\cos^2\alpha + \sin\beta + 2\sin\beta\cos^3\alpha}{2\sin\alpha(1+2\cos^3\alpha)}\,F_e^{b} = N_0\right)$$

$$F_e^{(b)} = \frac{N_0}{\frac{\cos^2\alpha}{1+2\cos^3\alpha}\cos\beta + \frac{1}{2\sin\alpha}\sin\beta}$$

now, is $|N_1| < N_0$ when $N_2 = N_0$?

$$N_1^{(b)} = \frac{N_0\cos\beta/(1+2\cos^3\alpha)}{\frac{\cos^2\alpha}{1+2\cos^3\alpha}\cos\beta + \frac{1}{2\sin\alpha}\sin\beta}$$

$$= \frac{N_0}{\cos^2\alpha + \frac{1+2\cos^3\alpha}{2\sin\alpha}\tan\beta}$$ $< N_0$ if Denom >

i.e., if $\cos^2\alpha + \dfrac{1+2\cos^3\alpha}{2\sin\alpha}\tan\beta > 1$

$$\frac{(1+2\cos^3\alpha)\tan\beta}{2\sin\alpha} > 1-\cos^2\alpha = \sin^2\alpha$$

$$\text{or} \quad \tan\beta > \frac{2\sin^3\alpha}{1+2\cos^3\alpha}$$

Summary:

a) N_1 will yield first, when $F_c^{(a)} = \dfrac{1+2\cos^3\alpha}{\cos\beta} N_1$

$$\text{if} \quad \tan\beta < \frac{2\sin^3\alpha}{1+2\cos^3\alpha}$$

b) N_2 will yield first when $F_0^{(b)} = \dfrac{N_0}{\dfrac{\cos^2\alpha}{1+2\cos^3\alpha}\cos\beta + \dfrac{\sin\beta}{2\sin\alpha}}$

$$\text{if} \quad \tan\beta > \frac{2\sin^3\alpha}{1+2\cos^3\alpha}$$

AT Collapse

a) $N_1 = N_2 = N_0$ or b) $N_2 = N_0 \quad N_3 = -N_0$

1 & 2 pl in Tension — 2 pl in T, 3 pl in C

for a)

equil $\rightarrow$ $F \sin\beta = N_0 \sin\alpha - N_3 \sin\alpha$

$$F \cos\beta = N_0 + N_0 \cos\alpha + N_3 \cos\alpha$$

elim N_3:

$$F \sin\beta \cos\alpha = N_0 \sin\alpha \cos\alpha - N_3 \underline{\sin\alpha\cos\alpha}$$

$$F \cos\beta \sin\alpha = N_0 \sin\alpha + N_0 \cos\alpha \sin\alpha + N_3 \underline{\cos\alpha \sin\alpha}$$

$$F(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = (1 + 2\cos\alpha)\sin\alpha \, N_0$$

$$F_0^{(a)} = \frac{(1+2\cos\alpha)\sin\alpha}{\sin(\alpha+\beta)} N_0$$

$$N_3 = \frac{\sin(\alpha-\beta) - \sin\beta}{\sin(\alpha+\beta)} N_0$$

OK iff $N_3 \geq -N_0 \rightarrow \sin\alpha\cos\beta - \cancel{\cos\beta\sin\alpha} - \sin\beta \geq -\sin\alpha\cos\beta - \cancel{\cos\beta}$

or $\tan\beta \leq 2\sin\alpha$

if b) : $N_2 = N_0 \qquad N_3 = -N_0$

equil : $F \sin\beta = 2 N_0 \sin\alpha$

$F \cos\beta = N_1$

$$F_0^{(b)} = \frac{2\sin\alpha}{\sin\beta} N_0$$

$$N_1 = \frac{2\sin\alpha}{\tan\beta} N_0$$

valid iff $N_1 \leq N_0$

$$\rightarrow \quad \frac{2\sin\alpha}{\tan\beta} \leq 1$$

or $\tan\beta \geq 2\sin\alpha$

comparing

$$F_0^{(a)} \quad \mathrm{I} \quad F_0^{(b)}$$

$$\frac{(1+2\cos\alpha)\sin\alpha}{\sin(\alpha+\beta)} N_0 \quad \mathrm{I} \quad \frac{2\sin\alpha}{\sin\beta} N_0$$

$$\sin\beta + 2\sin\beta\cos\alpha \quad \mathrm{I} \quad 2\sin\alpha\cos\beta + 2\sin\beta\cos\alpha$$

$$\tan\beta \quad \mathrm{I} \quad 2\sin\alpha$$

$\therefore$ $F_0^{(a)} < F_0^{(b)}$ if $\tan\beta < 2\sin\alpha$

$F_0^{(b)} < F_0^{(a)}$ if $\tan\beta > 2\sin\alpha$