Notes Lim Anal Thms

Set up the problem

~~Structure~~ $\mathcal{B}$ ~ set of loads : F Rigid/perf pl

determine max ρ ∋ ρF is supported

Define

Yield Moment: $M_0 = M_0(X)$ max (pl) mom

Yield Hinge:

1) if $M < M_0$ YH trans M with no rotation

2) " $M = M_0$ " with rot of any mag

3 $M \ngtr M_0$

Mechanism: $\mathcal{B}$ is a mech if ∃ enuf YH in $\mathcal{B}$

to permit inf motion of all or part of $\mathcal{B}$

SAF (Stat Adm Field)

1) Mom in int + ext equil with ρF

2 all $M \leq M_0$

Safety Factor $\mathcal{S}$

1) Mom distr assoc with $\mathcal{S}F$ is SA

2) if YH inserted wherever $M = M_0$, $\mathcal{B} \to$ a mechanism

SAM : S^- is a SAM if $\exists$ at least one SAF in equil with S^-F

KAF : Kinematically Adm Field

Consider Mech : work of F on displ W^* is >0

let θ_k^* be rot of hinge k. Define M_k^* at k as M_0 if $\theta_k^* > 0$, $-M_0$ if $\theta_k^* < 0$.

asterisked qtys : KAF

KAM : Given KAF, the assoc KAM S^+ is defined $\ni$ internal work done by M^* on θ^* is equal to ext work done by S^+F on W^*

thus

$$S^+ = \frac{\sum M_k^* \theta_k^*}{\int F(x)\, W^*(x)\, dX}$$

Lower Bound Theorem : $\mathcal{S}F$ is the largest SAM

Upper " " $\mathcal{S}F$ " smallest KAM

ie $\mathcal{S}^- \le \mathcal{S} \le \mathcal{S}^+$

Lower Bound Theorem

let M° be a SAF in eq with $\delta^{-}F$

let M, θ, w be actual moments, rotations, displacement under loads δF

applying virt work (twice)

$$\sum M_k \theta_k = \int \delta F w \, dx$$

$$\sum M_k^{\circ} \theta_k = \int \delta^{-} F w \, dx$$

$$\therefore \quad (\delta - \delta^{-}) \int F w \, dx = \sum (M_k - M_k^{\circ}) \theta_k$$

now if $\theta_k > 0$, $M_k = M_0$ and $M_k^{\circ} \leq M_0$

$$\therefore \quad (M_k - M_k^{\circ}) \theta_k \geq 0$$

also if $\theta_k < 0$, $M_k = -M_0$ and $M_k^{\circ} \geq -M_0$

$$\therefore \quad M_k - M_k^{\circ} \leq 0$$

but $(M_k - M_k^{\circ}) \theta_k \geq 0$

$$\therefore \quad \Sigma (M_k - M_k^\circ)\,\theta_k \geq 0$$

now $\int F W\,dx$ is ~~prop~~ proportional to actual work of collapse it is pos

$$\therefore \quad \lambda \geq \lambda^-$$

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let M^*, θ^*, W^* be moments, rotations + displ in a KAF with KAM λ^+.

let M be actual moments in equil with λF

$$\text{virt work:} \quad \Sigma M_k \theta_k^* = \int \lambda F W^* dx$$

$$\text{definition} \quad \Sigma M_k^* \theta_k^* = \lambda^+ \int F W^* dx$$

$$\therefore \quad (\lambda^+ - \lambda) \underbrace{\int F W^* dx}_{>0} = \underbrace{\Sigma (M_k^* - M_k)\theta_k^*}_{\geq 0}$$

$$\therefore \quad \lambda^+ \geq \lambda$$