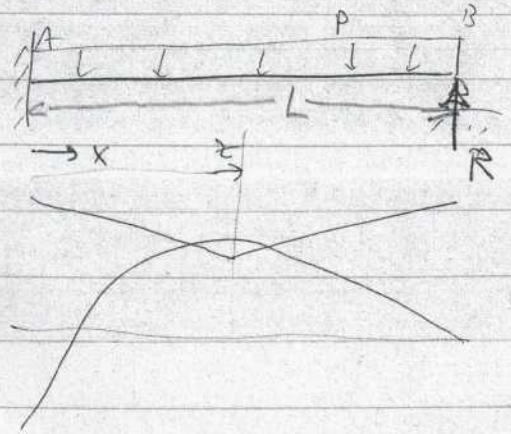
Example of Limit Analysis



$$M(x) = R(L-x) - \tfrac{1}{2}P(L-x)^2$$

at collapse, hinges at $X = 0$, Z ↑ unknown

now $-M_0 \le M \le M_0$

and since M contin, $\exists$ rel max at $X = Z$

and $M(0) = RL - \tfrac{1}{2}PL^2 = -M_0$ (1)

$$M(z) = R(L-z) - \tfrac{1}{2}P(L-z)^2 = M_0$$

$$M'(z) = -R + P(L-z) = 0$$

$$\Rightarrow \quad R = \underset{4.82843}{\left(2+2\sqrt{2}\right)}\frac{M_0}{L}$$

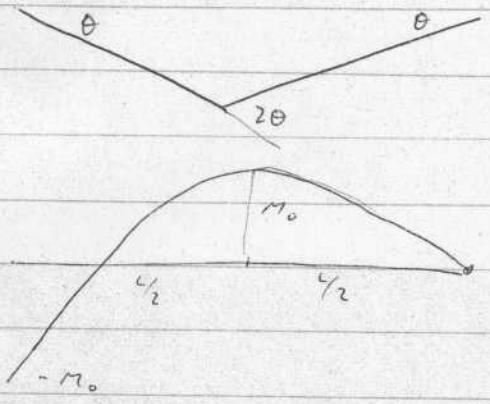
$$P = \underset{11.65685}{\left(6+4\sqrt{2}\right)}\frac{M_0}{L^2}$$

$$z = \underset{0.58579}{\left(2-\sqrt{2}\right)}L$$

Using LA th

note hinge angulars form mechanism

take $z = \frac{L}{2}$



$$W_i = 3 M_o \theta$$

$$\Delta_{max} = \frac{L}{2}\theta$$

$$\Delta_{avg} = \frac{L\theta}{4}$$

$$W_e = \frac{1}{4} P L^2 \theta$$

for $W_i = W_e$: $\quad 3M_o = \frac{1}{4} P^+ L^2$

or $P^+ = 12 M_o / L^2$ an upper bd

can find moment dist (max at $\frac{1}{2}$) $= M_o$

$$M = R(L-x) - \frac{12 M_o}{2L^2}(L-x)^2$$

$$= 5\frac{M_o}{2}(L-x) - 6\frac{M_o}{L^2}(L-x)^2$$

from ① $\quad RL - \frac{1}{2}PL^2 = -M_o$

$\rightarrow R = 5\frac{M_o}{L}$

max @ $x = \frac{1}{2}$ $\quad > M_o$ near $x = \frac{1}{2}$

$$M\left(\tfrac{1}{2}\right) = \frac{5}{2}M_o - \frac{6M_o}{4}$$

$$M = \frac{M_0}{L}\left(5(L-x) - \frac{6}{L}(L-x)^2\right)$$

$$M' = \frac{M_0}{L}\left[-5 + \frac{12}{L}(L-x)\right] = 0$$

$$-5 + 12 = 12\frac{x}{L}$$

$$x = \frac{7}{12}L$$

$$M = \frac{M_0}{L}\left(5\left(\frac{5}{12}\right)L - \frac{6}{L}\left(\frac{25}{144}\right)L^2\right)$$

$$= \frac{50 - 25}{24} M_0 = \frac{25}{24} M_0$$

thus for $P^+ = 12\frac{M_0}{L^2}$ $\quad M_{max} = \frac{25}{24} M_0$

now for $\frac{24}{25} P^+ = \left(\frac{24}{25} \cdot 12\right)\frac{M_0}{L^2}$ $\quad M_{max} = M_0$

+ hence

$$\frac{PL^2}{M_0} = \frac{24 \cdot 12}{25} = \frac{288}{25} = \frac{1152}{100} = 11.52 \text{ is a lb}$$

$$\therefore P^- = 11.52$$

$$\therefore 11.52 \leq \frac{PL^2}{M_0} \leq 12$$

$$\text{or } \frac{PL^2}{M_0} = 11.76 \pm .24$$

on left go around (hinge @ $x = \frac{7}{12}L$)

$$\mathcal{H}_i = 17 M_0 \theta \qquad \mathcal{H}_e = \frac{35}{24} P L^2 \theta$$

$$p^+ = \frac{408}{35} = 11.65714$$

$$M_{max} \text{ @ } x = \frac{239}{408} = .58578$$

$$\frac{M}{M_0} = \frac{28,561}{28560}$$

$$\therefore \qquad p^- = 11.65674$$