$$T_{jk}\,\epsilon_{ijk} = t_i$$

mult by ϵ_{imn}

$$T_{jk}\,\underbrace{\epsilon_{ijk}\,\epsilon_{imn}} = t_i\,\epsilon_{imn}$$

$$T_{jk}\left(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}\right) = t_i\,\epsilon_{imn}$$

$$T_{mn} - T_{nm} = 2T_{mn} = t_i\,\epsilon_{imn}$$ if T_{mn} is antisym

$$\therefore\quad T_{mn} = \frac{1}{2}\,\epsilon_{imn}\,t_i$$

– – –

since prod of vectors is a tensor :

$u_j V_k$ a tensor if u_i, V_i are 2 vectors

hence $w_i = \epsilon_{ijk}\,u_j\,V_k$ is the dual vector of the tensor $u_j V_k$

consider components of w_i

$$w_1 = u_2V_3 - u_3V_2 \quad \text{etc}$$

hence w_i corresponds to cross product

Many cumbersome vector identities may be easily proven

eg: $\vec{T} \times (\vec{U} \times \vec{V})$

$$\vec{U} \times \vec{V} = \epsilon_{kmn} U_m V_n$$

$$\epsilon_{ijk} T_j \epsilon_{kmn} U_m V_n$$

$$= \epsilon_{ijk} \epsilon_{kmn} T_j U_m V_n$$

$$=$$

$$= \epsilon_{kij} \epsilon_{kmn} T_j U_m V_n$$

$$= (\delta_{im} \delta_{jn} - \delta_{in} \delta_{jm}) T_j U_m V_n$$

$$= T_j U_i V_j - T_j U_j V_i$$

$$= \vec{U} (\vec{T} \cdot \vec{V}) - \vec{V} (\vec{T} \cdot \vec{U})$$