MAE 443/543 Continuous Control
Motor Simulation

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Fall 2007

1 Introduction

During the course of this exercise you should get an insight on how to use the LabVIEW skills you acquired in previous exercise to develop a virtual model of an electric motor. At the end of this experiment you should be able to generate a VI to simulate the motor.

2 Mathematical Model

This section of the lab should be read over and completely understood before attending the lab. It is encouraged for the student to work through the derivations as well as to get a thorough understanding of the underlying mechanics. Given in Fig. 1 is the electrical schematic of the motor.

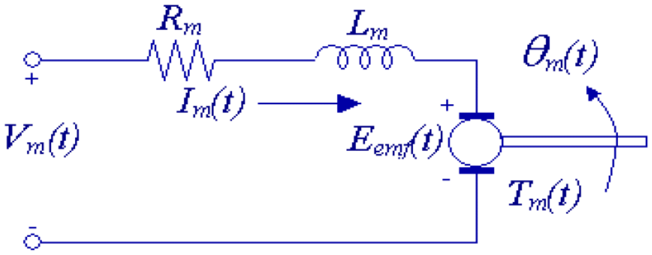

Figure 1: Electric Motor

When the armature is conducting current, (I_m), in the magnetic field, it produces a mechanical torque on the shaft proportional to the current. i.e.,

$$T_m = \eta_m K_t I_m \tag{1}$$

where η_m represents the motor efficiency and K_t is the motor-torque constant. As the armature rotates in the magnetic field it also generates a voltage proportional to the speed of rotation, known as the back-emf (E_{emf}).

$$E_{emf} = K_m \dot{\theta}_m \tag{2}$$

where K_m is the back-emf constant and $\dot{\theta}_m$ is the motor shaft velocity. Now using the Kirchhoff's voltage law, the following equation is obtained:

$$V_m - R_m I_m - L_m \frac{dI_m}{dt} - E_{emf} = 0 \tag{3}$$

Since $L_m \ll R_m$, we can disregard the motor inductance. i.e.,

$$\begin{align} I_m &= \frac{V_m - E_{emf}}{R_m} \tag{4} \\ &= \frac{V_m - K_m \dot{\theta}_m}{R_m} \tag{5} \end{align}$$

Now consider the mechanical aspect of the motor and applying the Newton's 2^{nd} law of motion we have

$$J_m \ddot{\theta}_m = T_m - \frac{T_l}{\eta_g K_g} \tag{6}$$

where J_m is the motor moment of inertia, η_g is the efficiency of the gearbox, K_g is the gear ratio, T_m is the torque generated by the motor, T_l is the torque applied at the load and the term $T_l/(\eta_g K_g)$ is the load torque seen though the gears. Now applying the Newton's 2^{nd} law at the load of the motor:

$$J_l \ddot{\theta}_l = T_l - B_{eq} \dot{\theta}_l \tag{7}$$

where $\dot{\theta}_l$ is the load shaft angular velocity, J_l is the load shaft moment of inertia, and B_{eq} is the viscous damping coefficient as seen at the output. Combining Eqs. (6) and (7) we have

$$J_l \ddot{\theta}_l = \eta_g K_g T_m - \eta_g K_g J_m \ddot{\theta}_m - B_{eq} \dot{\theta}_l \tag{8}$$

Notice that $\theta_m = K_g \theta_l$ and $T_m = \eta_m K_t I_m$, now the above equation can be rewritten as

$$J_l \ddot{\theta}_l = \eta_g K_g \eta_m K_t I_m - \eta_g K_g^2 J_m \ddot{\theta}_l - B_{eq} \dot{\theta}_l \tag{9}$$

Symbol	Description	Nominal Value SI units
B_{eq}	Viscous damping coefficient	4.0×10^{-3}
η_g	Gearbox efficiency	0.9
η_m	Motor efficiency	0.69
$J_{eq} = J_l + \eta_g J_m K_g^2$	Moment of inertia at the load	2.0×10^{-3}
J_m	Motor moment of inertia	3.87×10^{-7}
K_g	Gear ratio	$70(14 \times 5)$
K_m	Back-emf constant	7.67×10^{-3}
K_t	Motor-torque constant	7.67×10^{-3}
R_m	Armature resistance	2.6

Table 1: Motor Parameters

For a complete listing of the system parameters required for the modeling of the motor, refer to Table 1. Now combining the electrical and mechanical equations, we obtain our desired transfer function:

$$\frac{\theta_l(s)}{V_m(s)} = \frac{b}{a_2 s^2 + a_1 s + a_0} \tag{10}$$

where b, a_2, a_1, and a_0 are functions of system parameters. It will be your pre-lab assignment to findout the exact representation of the transfer function using the motor parameters given in Table 1.

3 Simulating a Motor

This section of the handout explains how one could numerically simulate the mathematic model of the motor that was analyzed earlier. During the pervious exercise we have already constructed a VI that would generate a signal based on user specifications. This user specified signal will be used as the reference signal into the motor. Therefore, the easiest way to complete this assignment would be to include a subVI to the existing VI and also have the user specify the system parameters. The modified VI should have 8 control items on the front panel, one for each of the following:

1. Signal type
2. Sampling time
3. Signal frequency

4. Signal amplitude

5. Four system parameters: b, a_2, a_1, and a_0

As the number of control items increase, it will be more difficult to keep track of each item. One way to get around this problem is by clustering the items based on some pattern. For example, all the system parameters can be clustered together in this case. Clustering of the control items can be done using a new control item called `Cluster`. `Cluster` can be obtained from `Modern` → `Array, Matrix & Cluster` of the Controls Palette. After adding the cluster into the front panel, you could include the four numeric controllers corresponding to the system parameters into the cluster.

Each cluster will have a single output and if you would like to access the individual data, you need to use a new function called `Unbundle by Name`. `Unbundle by Name` can be obtained from `Programming` → `Cluster, Class, & Variant` of the Functions Palette. After you wire the node to an input cluster, right-click the name terminals to select elements from the shortcut menu. You also can use the Operating tool to click the name terminals and select from a list of cluster elements. Also note that the unbundled items can be clustered back together using a function called `Bundle by Name`.

The clusters will come in handy while you construct your subVI for the motor model. This subVI should have 8 inputs, b, a_2, a_1, a_0, dt, reference signal, position, and velocity. The output of this subVI would be the position and the velocity. It is safe to assume zero initial velocity and position. Using the inputs, the subVI should calculate the angular acceleration and by integrating the acceleration, the angular position and velocity could be determined. Integration can be implemented in the motor-subVI (or inside a new subVI) using the finite difference scheme given below:

$$x(n) \approx x(n-1) + \dot{x}(n) \triangle t \tag{11}$$

The integrated signals are then feedback into the same subVI using a `Feedback Node`. When you connect the output of the subVI to its input the Feedback Node should automatically appear. A complete block diagram for the subVI and the main VI are given in Figs. 2 and 3.

Once you have your VIs completed, conduct numerous simulations with different signals. Make sure you include necessary documentation in your block diagram and save all your work for future reference.

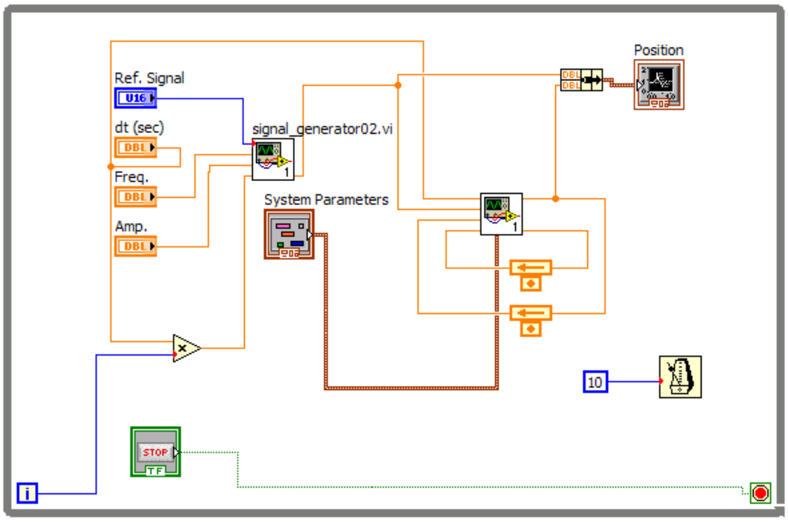

Figure 2: Main VI for the Motor Simulation

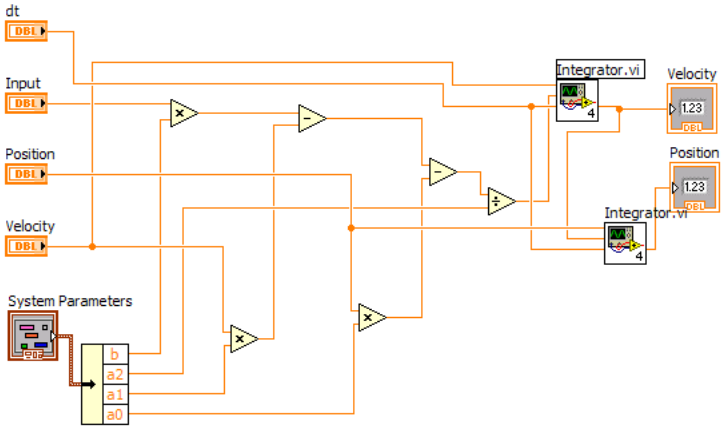

Figure 3: subVI for the Motor Simulation